Lecture Note

Fluid Mechanics

(part-I)

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Course Content

Ch1. Fluid properties
Ch2. Fluid Statics
Ch3. Fluid Flow Concepts and Equations
Ch4. Viscous Effects and Flow Resistances
Ch5. Compressible Flow
Ch6. Ideal Fluid Flow
Ch7. Flow in Closed Conduits and Open Channels

Text Book: Fluid Mechanics by Donald F. Young and others (1997)

Reference Books

1. Fluid Mechanics with Engineering application by Robert L. Daugherty and others. (1989)
2. Fluid Mechanics for Civil Engineers by Webber.
3. Fluid Mechanics by Streeter
Exams: First Term Exam+ Term paper : 10% Mid Term Exam:20% Second Term Exam + Term paper : 10 % Final Exam: 50 % Laboratory : 10 %

Terminology

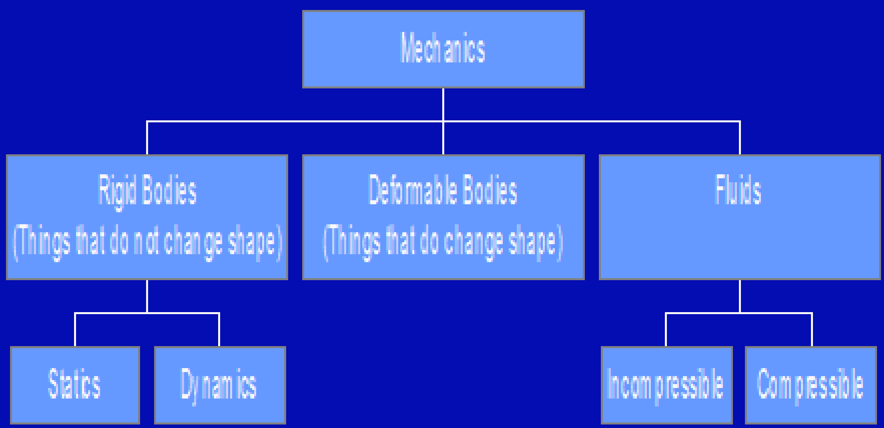
Variable	Symbol	unit	المتغير
Density	p Roh	Kg/m^3	الكثافة
Specific Wieght	ɤ Gamma	N/m^3 For water=9.81 N/m^3	الوزن النوعي
Specific gravity	s.g	dimensionless	الكثافة النسبية
Viscosity	µ mu	N.s/m^2	اللزوجة
Shear stress	Ţ Tao	N/m2	جهد القص
Surface tension	σ segmma	N/m	الشد السطحي
Force	F	N	القوة
Pressure	P	N/m^2	الضغط
velocity	V	m/sec	السرعة
Discharge	Q	M^3/sec	التصريف
Acceleration	a	m/s^2	التعجيل
Gravity Acceleration	g	m/s^2 (9.81)	التعجيل الارضي
Work	W	N.m	الشغل
Power	Po	N.m/s	الطاقة
Torque	T	N.m	عزم اللي

Fluid properties

Introduction

- Physical Characteristics of Fluids
- Distinction between Solids, Liquids, and gases
- Significance of Fluid Mechanics

Branch of Mechanics



Physical Characteristics of Fluids

- Fluid mechanics is the science that deals with the action of forces on fluids.
- Fluid is a substance
- The particles of which easily move and change position
- That will continuously deform

Distinction Between Solids, Liquids & Gases

- A fluid can be either gas or liquid.
- Solid molecules are arranged in a specific lattice formation and their movement is restricted.
- Liquid molecules can move with respect to each other when a shearing force is applied.
- The spacing of the molecules of gases is much wider than that of either solids or liquids and it is also variable.

DIMENSIONS AND UNITS

UNIT SYSTEMS

- We will work with one unit systems in FLUID MECHANICS:
  - International System (SI)

- SI UNITS

In the SI system, the unit of force, the *Newton*, is derived unit.
The meter, second and kilogram are base units.

Basic Unit System & Units

The SI system consists of six **primary** units, from which all quantities may be described but in fluid mechanics we are generally only interested in the top four units from this table.

Quantity	SI Unit	Dimension
length	metre, m	L
mass	kilogram, kg	M
time	second, s	T
temperature	Kelvin, K	θ
current	ampere, *A*	I
luminosity	candela	Cd

Derived Units

There are many **derived** units all obtained from combination of the above **primary** units. Those most used are shown in the table below:

Derived Units

Quantity	SI Unit		Dimension
velocity	m/s	ms^{-1}	LT^{-1}
acceleration	m/s^2	ms^{-2}	LT^{-2}
force	N $kg\ m/s^2$	$kg\ ms^{-2}$	$M\ LT^{-2}$
energy (or work)	Joule J N m, $kg\ m^2/s^2$	$kg\ m^2s^{-2}$	ML^2T^{-2}
power	Watt W N m/s $kg\ m^2/s^3$	Nms^{-1} $kg\ m^2s^{-3}$	ML^2T^{-3}
pressure (or stress)	Pascal P, N/m^2, $kg/m/s^2$	Nm^{-2} $kg\ m^{-1}s^{-2}$	$ML^{-1}T^{-2}$
density	kg/m^3	$kg\ m^{-3}$	ML^{-3}
specific weight	N/m^3 $kg/m^2/s^2$	$kg\ m^{-2}s^{-2}$	$ML^{-2}T^{-2}$
relative density	a ratio no units		1 no dimension
viscosity	$N\ s/m^2$ kg/m s	$N\ sm^{-2}$ $kg\ m^{-1}s^{-1}$	$M\ L^{-1}T^{-1}$
surface tension	N/m $kg\ /s^2$	Nm^{-1} $kg\ s^{-2}$	MT^{-2}

SI System of Units

- The corresponding unit of force derived from Newton's second law:

" the force required to accelerate a kilogram at one meter per second per second is defined as the *Newton* (N)"

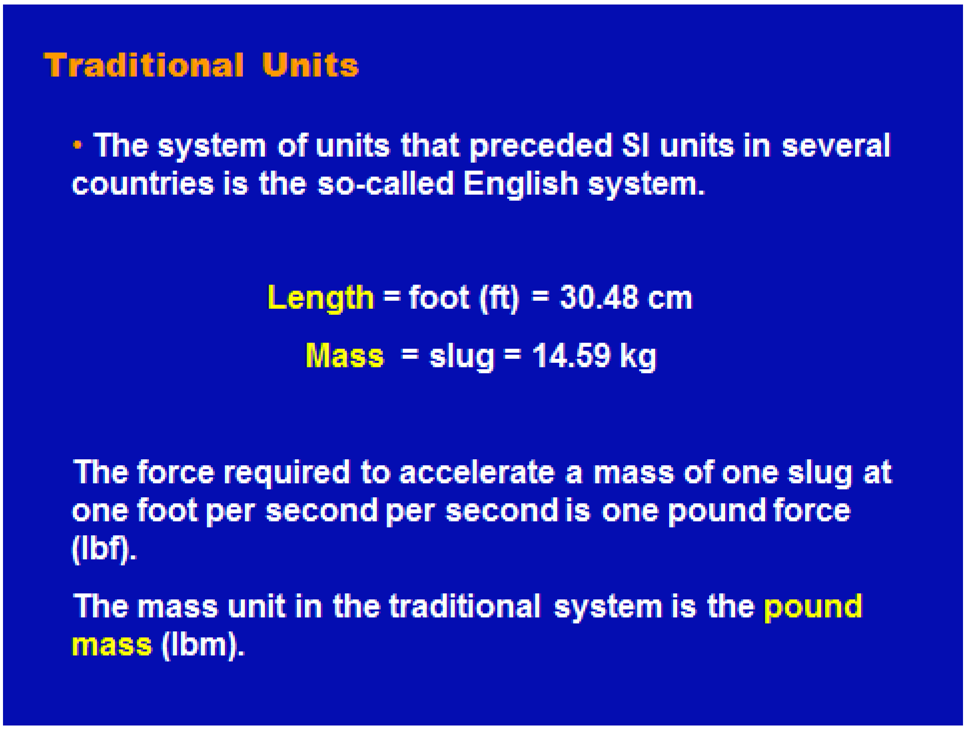
The acceleration due to gravity at the earth's surface: 9.81 m/s^2.

Thus, the *weight of one kilogram* at the earth's surface:

$$W = m\,g$$

$$= (1)\ (9.81)\ kg\ m\ /\ s^2$$

$$= 9.81\ N$$



Main units are: Length (L), Mass (M), and Time (T). Most of the other quantities like force, pressure, power, and more can be derived from the main three quantities: LMT.

In fluid mechanics, there are only four primary dimensions from which all the dimensions can be derived: mass, length, time, and force. The brackets around a symbol like [M] mean "the dimension" of mass. All other variables in fluid mechanics can be expressed in terms of [M], [L], [T], and [F].

For example, acceleration has the dimensions [LT-2]. Force [F] is directly related to mass, length, and time by Newton's second law,
Force= Mass x Acceleration

$F = m a$

From this we see that, dimensionally, [F] = [MLT-2].
1 kg-force = 9.81 Newton of force = 9.81 N

Table (1) Primary Dimensions in Fluid Mechanics in the International System of Units (SI)

Dimension	Unit
Length (L)	**meter (m)**
Mass (M)	**kilogram (kg)**
Time (T)	**second (s)**
Force (F)	**Newton (N) = kg.m/s2) = MLT-2**
Temperature (θ)	**Kelvin K °** **Celsius C °** K**=C° + 273°**

Table (2) Secondary Dimensions in Fluid Mechanics in the International System of Units (SI)

Area [L2]	**m2**
Volume [L3]	m3
Velocity [LT-1]	m/sec
Acceleration [LT-2]	m/sec2
Pressure or stress [FL-2]	Pa= N/m2(Pascal)
Angular Velocity	sec-1
Energy, work [FL]	J = Nm (Joule)
Power [FLT-1]	W = J/sec (Watt)
Specific mass (ρ) [ML-3]	kg/m3

Table (3) Commonly used prefixes for SI units:

Factor by which unit is multiplied	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n

Fluid Properties

FLUID PROPERTIES

Every fluid has certain characteristics by which its physical conditions may be described.

We call such characteristics as the fluid properties.

- **Specific Weight**
- **Mass Density**
- **Viscosity**
- **Vapor Pressure**
- **Surface tension**
- **Capillarity**
- **Bulk Modules of Elasticity**

Specific Weight, γ

The gravitational force per unit volume of fluid, or simply "weight per unit volume".

- Water at 20 °C has a specific weight of 9.79 kN/m^3.

Specific weight= γ = will be expressed in force-length-time dimensions and will have dimensions of force [F] per unit volume [L3].

$$Specific weight = \frac{Weight}{Volume}$$

$$[\gamma] = \left[\frac{F}{L^3}\right] = \left[\frac{M}{L^2T^2}\right], (kg/m^3)$$

Because the weight (a force), W, related to its mass, M, by Newton's second law of motion in the form

$W = Mg$

In which g is the acceleration due to the local force of gravity, specific weight and specific mass will be related by a similar equation,

$$\gamma = \rho g$$

Mass Density, ρ

Mass Density, ρ
The "mass per unit volume" is mass density. Hence it has units of kilograms per cubic meter.
- The mass density of water at 4 oC is 1000 kg/m3 while it is 1.20 kg/m3 for air at 20 oC at standard pressure.

- **Mass Density of a fluid is its mass per unit volume.**

$$\rho = \frac{Mass}{Volume} = \frac{m}{V}$$

Density of water:

SI Units:

$$\rho_w = 1000 \frac{kg}{m^3}$$

$$[\rho] = \left[\frac{M}{L^3}\right]$$

kg/m^3

Specific Gravity, S

- **The ratio of specific weight of a given liquid to the specific weight of water at a standard reference temperature (4 °C)is defined as *specific gravity*, S.**
- **The specific weight of water at atmospheric pressure is 9810 N/m^3.**
- **The specific gravity of mercury at 20 °C is**

$$S_{Hg} = \frac{133 \text{ kN/m}^3}{9.81 \text{ kN/m}^3} = 13.6$$

Specific Volume

Specific volume of a fluid is the volume occupied by a unit mass of fluid.

$$\text{Specific Volume, } v = \frac{volume}{mass} = \frac{V}{m} = \frac{1}{\rho}$$

m^3/kg

$$S = \frac{\rho}{\rho_{water}} = \frac{\rho/g}{\rho_{water}/g} = \frac{\gamma}{\gamma_{water}} \quad \text{(dimensionless)}$$

$$\text{Weight, } W = (S.G \times \gamma_w) \times V$$

$$S.G = \frac{W}{\gamma_w} \times \frac{1}{V} \qquad S.G = \text{constant} \times \frac{1}{V}$$

Which means that, the higher the specific gravity, the lower the volume of displaced liquid. Consequently, graduation on the stem corresponding to different depths of submergence of the hydrometer can be made to indicate directly the specific gravity of the liquid being measured.

Ideal Gas Law

A form of the general equation of state, relating pressure, specific volume, and temperature;

$$pV = nR_uT \;\longrightarrow\; p = \frac{nR_uT}{V} \;\longrightarrow\; p = \frac{nR_uT}{V}\frac{MW_{gas}}{MW_{gas}} \;\longrightarrow\; p = \rho RT$$

$$\frac{nMW_{gas}}{V} = \text{mass per unit vol.} = \text{density} \qquad \frac{R_u}{MW_{gas}} = \text{gas constant, R}$$

- p = absolute pressure [N/m2]
- V = volume [m3]
- n = number of moles
- R_u = universal gas constant
- [8.314 kJ/kmol-K; 0.287 kPa·m^3/kg ·K]
- T = absolute temperature [K]
- MW_{gas} = molecular weight of gas

VISCOSITY

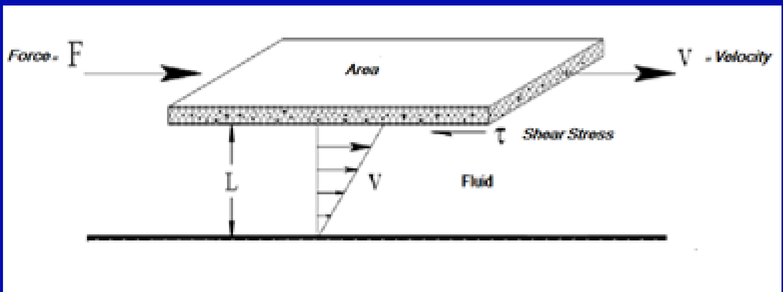
- **What is the definition of "strain"?**

  "Deformation of a physical body under the action of applied forces"
- **Solid:**
  - **shear stress applied is proportional to shear strain (proportionality factor: *shear modulus*)**
  - **Solid material ceases to deform when equilibrium is reached**
- **Liquid:**
  - **Shear stress applied is proportional to the time rate of strain (proportionality factor: *dynamic (absolute) viscosity*)**
  - **Liquid continues to deform as long as stress is applied**

Viscosity

Viscosity measures a fluids ability to resist shear stress.

Hypothetical Experiment:



Where:

v Velocity of the moving plate (varies linearly from 0 at the stationary surface to maximum at the contact surface between the moving plate and fluid).

$$\frac{v}{L} = \frac{\Delta v}{\Delta L}$$

A Contact area between the moving plate and fluid.

F Applied force to the moving plate.

L Thickness of fluid layer

$$F \propto A\frac{v}{L} \quad \rightarrow \quad F = \mu A\frac{v}{L}$$

Where μ is the proportional constant

Where F is the external applied force.

Summation of forces: $\Sigma F_x = 0$

Driving forces – Resisting forces = 0

$$F - F_R = 0 \rightarrow F = F_R = \tau A$$

$$\frac{F}{A} = \mu \frac{v}{L} \rightarrow \tau = \mu \frac{v}{L}$$

Assumptions:

1. Small gap thickness
2. v is not too large
3. Slope of the velocity distribution (assuming linear distribution)

$$\text{as } \Delta L \rightarrow 0, \frac{\Delta v}{\Delta L} \rightarrow \frac{dv}{dy}$$

$$\tau = \mu \frac{dv}{dy}$$

Viscosity of the fluid μ = Shear stress / velocity gradient $\mu = \tau / \frac{dv}{dy}$

Where:

τ Shear stress (N/m2)

μ Absolute (dynamic) viscosity, which measures ability of a fluid to flow

The units of μ are "Pa.s = $\frac{N.s}{m^2} = \frac{kg.m}{s^2}\frac{s}{m^2} = \frac{kg}{m.s}$"

Kinematic Viscosity = Absolute Viscosity/ density

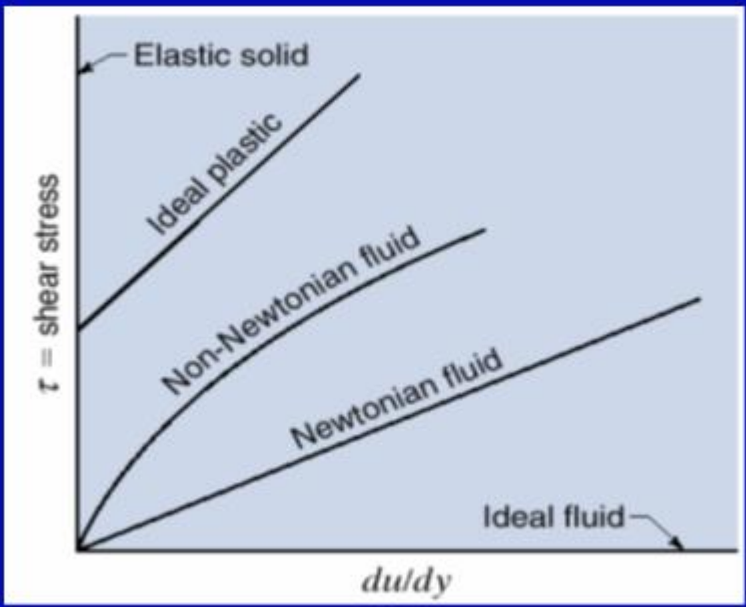
$$\nu = \frac{\mu}{\rho} = \frac{N.s}{m^2} / \frac{kg}{m^3} = \frac{kg.m.s}{s^2 m^2}\frac{m^3}{kg} = \frac{m^2}{s}(stoke)$$

A fluid for which the constant of proportionality (i.e., the absolute viscosity, μ) does not change with rate of deformation is called a Newtonian fluid,

Newtonian fluid, and this plots as a straight line in the following figure.
The slope of this line is the absolute viscosity, μ. The equation of a straight line, $y = mx$ is similar to Newton's law of viscosity,

$$\tau = \mu \frac{dv}{dy}$$

The ideal fluid, with no viscosity (μ = 0), falls on the horizontal axis, while the true elastic solid plots along the vertical axis.
A plastic that sustains a certain amount of stress before suffering a plastic flow corresponds to a straight line intersecting the vertical axis at the yield stress.
There are certain **non-Newtonian fluids** in which μ varies with the rate of deformation. These are relatively uncommon in engineering usage.
Typical non-Newtonian fluids include paints, printer's ink, gels and emulsions, sludge and slurries, and certain plastics.



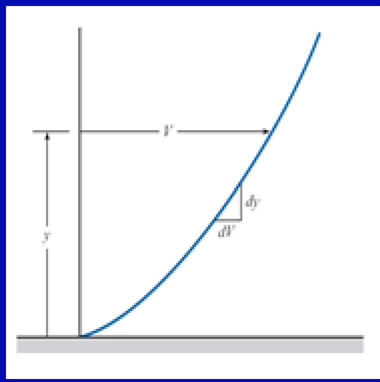
Example of the effect of viscosity

- Think: resistance to flow.
- *V* : fluid velocity
- *y* : distance from solid surface
- Rate of strain, dV/dy
- μ : dynamic viscosity [N.s/m2]

τ: shear stress

Shear stress: An applied force per unit area needed to produce deformation in a fluid

$$\tau = \mu \, dV/dy$$



Velocity distribution next to a boundary

VISCOSITY μ

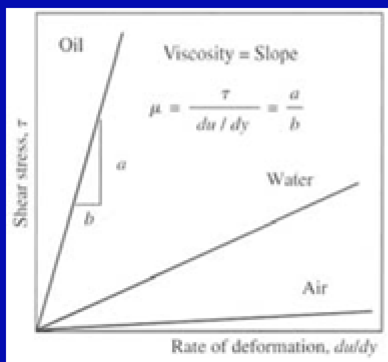
- Would it be easier to walk through a 1-m pool of water or oil?
  - *Water*

Why?

- *Less friction in the water*

- *Rate of deformation*
  - Water moves out of your way at a quick rate when you apply a shear stress (*i.e.*, walk through it)
  - Oil moves out of your way more slowly when you apply the same shear stress

$\tau = \mu \, dV/dy$



Viscosity is:

- slope of the line shown above
- the ratio between shear stress applied and rate of deformation

Kinematic Viscosity

- Many fluid mechanics equations contain the variables of
  - Viscosity, μ
  - Density, ρ

So, to simplify these equations sometimes use *kinematic viscosity* (ν)

$$\nu = \frac{\mu}{\rho} = \frac{N.s/m^2}{kg/m^3} = m^2/s$$

Terminology

Viscosity, μ

Absolute viscosity, μ

Dynamic viscosity, μ

Kinematic Viscosity, ν

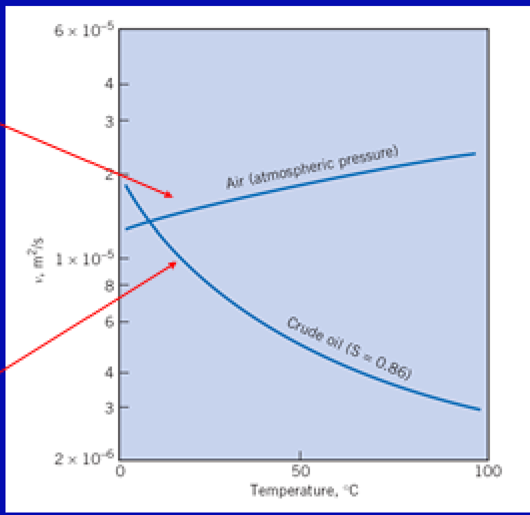
Other viscosity highlights

- Viscous resistance is independent of the pressure in the fluid.
- Viscosity is a result of molecular forces within a fluid.
- For liquid, cohesive forces decrease with increasing temperature → decreasing μ
- For gas, increasing temperature → increased molecular activity & shear stress: increasing μ

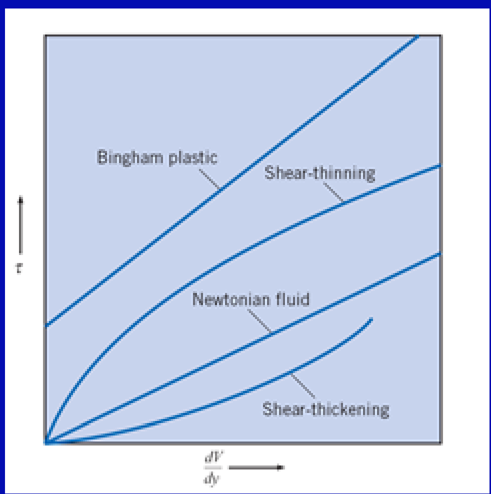
Kinematic viscosity for air & crude oil

Increasing temp → increasing viscosity

Increasing temp → decreasing viscosity

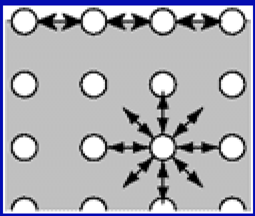


Newtonian vs. Non-Newtonian Fluids



- *Newtonian fluid*: shear stress is proportional to shear strain
  - Slope of line is dynamic viscosity
- *Shear thinning*: ratio of shear stress to shear strain decreases as shear strain increases (toothpaste, catsup, paint, etc.)
- *Shear thickening*: viscosity increases with shear rate (glass particles in water, gypsum-water mixtures).

Surface Tension



Fluid	Surface Tension σ_s, N/m*
Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

* Multiply by 0.06852 to convert to lbf/ft.

- A molecules in the interior of a liquid is under attractive force in all direction.
- However, a molecule at the surface of a liquid is acted on by a net inward cohesive force that is perpendicular to the surface.
- Hence it requires work to move molecules to the surface against this opposing force and surface molecules have more energy than interior ones
- Higher forces of attraction at surface
- Creates a "stretched membrane effect"

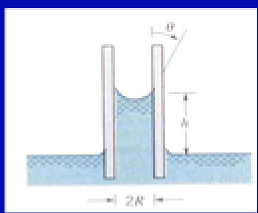
Capillarity

Rise and fall of liquid in a capillary tube is caused by surface tension. Capillarity depends on the relative magnitudes of the cohesion of the liquid to walls of the containing vessel.

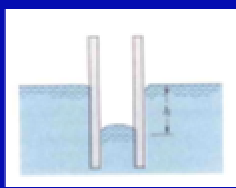
When the adhesive forces between liquid and solid are larger than the liquid's cohesive forces, the meniscus in a small diameter tube will tend to be concave

If adhesive forces are smaller than cohesive forces the meniscus will tend to be convex, for example mercury in glass.

concave



water



mercury

convex

Differences between adhesive & Cohesive

A distinction is usually made between an adhesive force, which acts to hold two separate bodies together (or to stick one body to another)

and

a cohesive force, which acts to hold together the like or unlike atoms, ions, or molecules of a single body.

Capillary Effect

For a glass tube in a liquid...

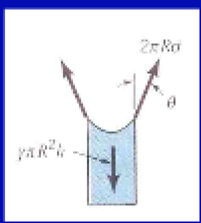
h=height of capillary rise (or depression)

σ=surface tension

θ=wetting angle

ɤ=specific weight

R=radius of tube



$$F_{\sigma,z} - W = 0$$

$$2\pi R \sigma Cos\,\theta = \pi R^2 h \gamma$$

$$h = \frac{2\sigma Cos\theta}{\gamma r}$$

If the tube is clean, θ° is 0 for water

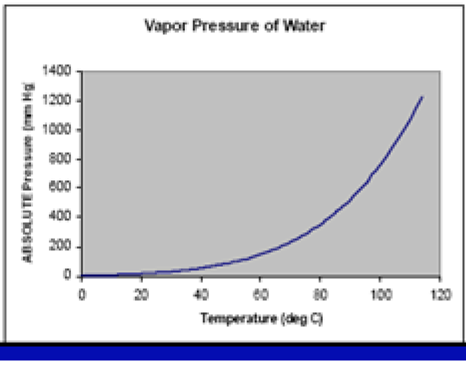
Vapor Pressure

Vapor pressure: the pressure at which a liquid will boil.

- Vapor pressure ↑ when temperature increases
- At atmospheric pressure, water at 100 °C will boil
- Water can boil at lower temperatures if the pressure is lower

When vapor pressure > the liquid's actual pressure

Temp. (°C)	Pressure (mm Hg)
-10	2.1
0	4.6
20	17.5
40	55.3
60	149.4
80	355.1
100	760
110	1075



Problem : 2 Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight (w)
$$= \frac{\text{Weight}}{\text{Volume}} = \frac{7\text{N}}{\left(\frac{1}{1000}\right)\text{m}^3} = \mathbf{7000 \text{ N/m}^3.\ Ans.}$$

(*ii*) Density (ρ)
$$= \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5 \text{ kg/m}^3.\ Ans.}$$

(*iii*) Specific gravity
$$= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because \text{Density of water} = 1000 \text{ kg/m}^3\}$$

$$= \mathbf{0.7135.\ Ans.}$$

Problem 3 Calculate the density, specific weight and weight of one liter of petrol of specific gravity 0.7

Solution. Given : Volume = 1 litre = $1 \times 1000\ cm^3 = \frac{1000}{10^6}\ m^3 = 0.001\ m^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Density (ρ) $= S \times 1000\ kg/m^3 = 0.7 \times 1000 = \mathbf{700\ kg/m^3}$. **Ans.**

(*ii*) *Specific weight* (w)

$w = \rho \times g = 700 \times 9.81\ N/m^3 = \mathbf{6867\ N/m^3}$. **Ans.**

(*iii*) *Weight* (W)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $w = \frac{W}{0.001}$ or $6867 = \frac{W}{0.001}$

$\therefore$ $W = 6867 \times 0.001 = \mathbf{6.867\ N}$. **Ans.**

Problem 4 : If the velocity distribution over a plate is given by

$$u = \frac{2}{3} y - y^2$$

in which u is the velocity in meter per second at a distance y meter above the plate. Determine the shear stress at y=0 and y=0.15 m. Take dynamic viscosity of fluid as 8.63 poises.

Solution. Given : $u = \frac{2}{3}y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{at\ y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{at\ y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

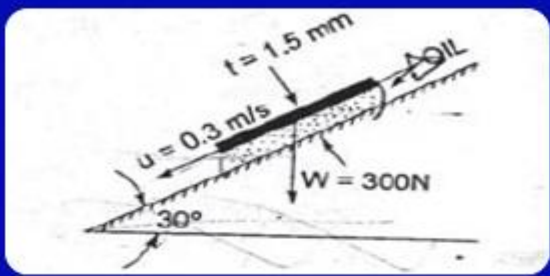
(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2.\ Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2.\ Ans.}$$

Proplem 5 : Calculate the dynamic viscosity of an oil , which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 30 degree as shown in figure below . The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.



Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300 \text{ N}$

Velocity of plate, $u = 0.3 \text{ m/s}$

Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane $= W \cos 60° = 300 \cos 60° = 150 \text{ N}$

Thus the shear force, F, on the bottom surface of the plate $= 150 \text{ N}$

and shear stress, $$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity $= u - 0 = u = 0.3 \text{ m/s}$

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \mathbf{11.7 \text{ poise. Ans.}}$$

Home Work due to Sunday 25-10-2015

Problem: 1

Two horizontal plates are placed 1.25 cm apart, The space between them being with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.

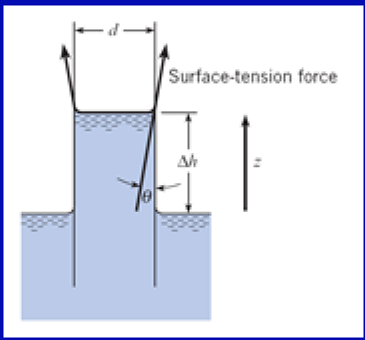
Problem :2

The space between two square flat plates is filled with oil. Each side of the plate is 60 cm. The thickness of oil film is 12.5 mm. The upper plate , which moves at 2.5 meter per esc requires a force of 98.1 N to Maintain the speed. Determine :

1)The dynamic viscosity of the oil in poise , and

2)The Kinematic viscosity of the oil in stokes if the Specific gravity of the oil is 0.95.

Problem 3: Capillary Rise Problem



$\sigma = 0.073$ N/m

- How high will water rise in a glass tube if the inside diameter is 1.6 mm and the water temperature is 20°C?

Answer: 18.6 mm

- *Hint: θ for water against glass is so small it can be assumed to be 0.*

Extra solved problems

Problem 6:
Find the kinematic viscosity of an oil having density 981 kg /m3. The Shear stress at a point in oil is 0.2452 N/m2 and velocity gradient at that point is 0.2 per second.

Solution. Given :

Mass density, $\rho = 981$ kg/m^3

Shear stress, $\tau = 0.2452$ N/m^2

Velocity gradient, $\frac{du}{dy} = 0.2$ s

Using the equation (1.2), $\tau = \mu \frac{du}{dy}$ or $0.2452 = \mu \times 0.2$

$\therefore$ $\mu = \frac{0.245}{0.200} = 1.226$ Ns/m^2

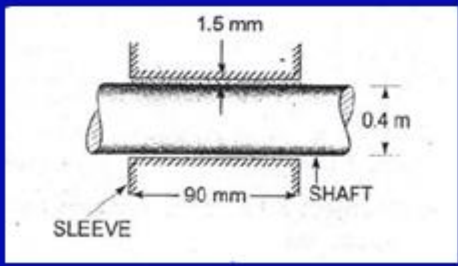
Kinematic viscosity ν is given by

$\therefore$ $\nu = \frac{\mu}{\rho} = \frac{1.226}{981} = .125 \times 10^{-2}$ m^2/sec

$= 0.125 \times 10^{-2} \times 10^4$ cm^2/s $= 0.125 \times 10^2$ cm^2/s

$= 12.5$ cm^2/s $=$ **12.5 stoke. Ans.** $(\because$ cm^2/s = stoke)

Problem 7: The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the Power lost in the bearing for a sleeve length of 90 mm. The thickness of the Oil film is 1.5 mm.



Solution. Given :

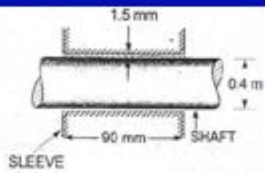


Fig. 1.5

Viscosity $\mu = 6$ poise

$$= \frac{6}{10}\frac{\text{Ns}}{\text{m}^2} = 0.6\ \frac{\text{Ns}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90$ mm $= 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5$ mm $= 1.5 \times 10^{-3}$ m

Tangential velocity of shaft, $u = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \frac{du}{dy}$

where du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

$$\tau = \overset{6}{10} \times \frac{3.98}{1.5 \times 10^{-3}} = 1592\ \text{N/m}^2$$

This is shear stress on shaft

∴ Shear force on the shaft, F = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05\ \text{N}$$

Torque on the shaft, $T = \text{Force} \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01$ Nm

∴ *Power lost $= \frac{2\pi NT}{60} = \frac{2\pi \times 190 \times 36.01}{60} =$ **716.48 W. Ans.**

Problem 8: A 15 cm diameter vertical cylinder rotates concentrically Inside another cylinder of diameter 15.10 cm. Both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 rpm. Determine the viscosity of fluid

Solution:

Given :

Diameter of cylinder = 15 cm= 0.15 m

Diameter of outer cylinder = 15.10cm= 0.151 m

length of cylinder= L= 25 cm= 0.25 m

Torque = T= 12.0 Nm

Speed = N= 100 r.p.m

Speed, $N = 100$ r.p.m.

Let the viscosity $= \mu$

Tangential velocity of cylinder, $u = \dfrac{\pi D N}{60} = \dfrac{\pi \times 0.15 \times 100}{60} = 0.7854$ m/s

Surface area of cylinder, $A = \pi D \times L = \pi \times 0.15 \times 0.25 = .1178\ \text{m}^2$

Now using relation $\tau = \mu \dfrac{du}{dy}$

where $du = u - 0 = u = .7854$ m/s

$$dy = \frac{0.151 - 0.150}{2}\ \text{m} = .0005\ \text{m}$$

$$\tau = \frac{\mu \times .7854}{.0005}$$

$\therefore$ Shear force, $F = \text{Shear stress} \times \text{Area} = \dfrac{\mu \times .7854}{.0005} \times .1178$

$\therefore$ Torque, $T = F \times \dfrac{D}{2}$

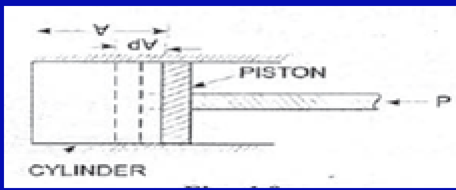
$$12.0 = \frac{\mu \times .7854}{.0005} \times .1178 \times \frac{.15}{2}$$

$\therefore$ $\mu = \dfrac{12.0 \times .0005 \times 2}{.7854 \times .1178 \times .15} = 0.864\ \text{N s/m}^2$

$= 0.864 \times 10 =$ **8.64 poise. Ans.**

Compressibility and Bulk Modulus

Compressibility is the reciprocal of bulk modulus of elasticity , K Which is defined as the ratio of compressive stress to volumetric strain.



Considered a cylinder fitted with a piston as shown in figure above
Let $\forall$ = volume of gas enclose in the cylinder
P= pressure of gas when volume is $\forall$

Let the pressure is increased to $p + dp$, the volume of gas decreases from $\forall$ to $\forall - d\forall$.

Then increase in pressure $= dp$ kgf/m^2

Decrease in volume $= d\forall$

$\therefore$ Volumetric strain $= -\dfrac{d\forall}{\forall}$

$-$ ve sign means the volume decreases with increase of pressure.

$\therefore$ Bulk modulus $K = \dfrac{\text{Increase of pressure}}{\text{Volumetric strain}}$

$$= \frac{dp}{\frac{-d\forall}{\forall}} = \frac{-dp}{d\forall}\forall$$

Compressibility is given by $= \dfrac{1}{K}$

Example:

Determine the bulk modulus of elasticity of a liquid , if the pressure of the liquid Is increase from 70 N/cm2 to 130 N/cm2. The volume of liquid decrease by 0.15 percent

Solution. Given :

Initial pressure $= 70\ \text{N/cm}^2$

Final pressure $= 130\ \text{N/cm}^2$

$\therefore$ dp = Increase in pressure $= 130 - 70 = 60\ \text{N/cm}^2$

Decrease in volume $= 0.15\%$

$$\therefore \quad -\frac{d\forall}{\forall} = +\frac{0.15}{100}$$

Bulk modulus, K is given by equation $K = \dfrac{dp}{-\dfrac{d\forall}{\forall}} = \dfrac{60\ \text{N/cm}^2}{\dfrac{.15}{100}} = \dfrac{60 \times 100}{.15} = 4 \times 10^4\ \text{N/cm}^2$. Ans.

Example 2:

What is the bulk modulus of elasticity of a liquid which is compressed In a cylinder from a volume of 0.0125 m3 at 80 N/cm2 pressure to a volume 0f 0.0124 m3 at 150 N/cm2 pressure?

Solution. Given :

Initial volume, $\forall = 0.0125\ \text{m}^3$

Final volume $= 0.0124\ \text{m}^3$

$\therefore$ Decrease in volume, $d\forall = .0125 - .0124 = .0001\ \text{m}^3$

$$\therefore \quad -\frac{d\forall}{\forall} = \frac{.0001}{.0125}$$

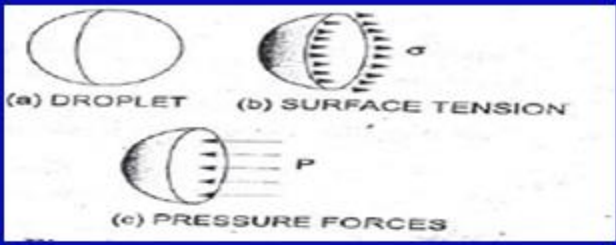
Initial pressure $= 80\ \text{N/cm}^2$

Final pressure $= 150\ \text{N/cm}^2$

$\therefore$ Increase in pressure, $dp = (150 - 80) = 70\ \text{N/cm}^2$

$$K = \frac{dp}{-\dfrac{d\forall}{\forall}} = \frac{70}{\dfrac{.0001}{.0125}} = 70 \times 125\ \text{N/cm} = 8.75 \times 10^3\ \text{N/cm}^2.\ \text{Ans.}$$

Surface Tension and liquid Droplet



(*i*) tensile force due to surface tension acting around the circumference of the cut portion

$$= \sigma \times \text{Circumference}$$
$$= \sigma \times \pi d$$

(*ii*) pressure force on the area $\frac{\pi}{4} d^2$ and $= p \times \frac{\pi}{4} d^2$

These two forces are equal:

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d}$$

Surface Tension on a Hollow Bubble like a soap bubble has two surfaces in contact with air. One inside and the other is outside. Thus the two surfaces are subjected to surface tension.

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$p = \frac{2\sigma\pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$

Example *Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.*

Solution. Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3}$ m

Pressure in excess of outside, $p = 2.5$ N/m²

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \mathbf{0.0125 \text{ N/m. Ans.}}$$

Example: Calculate the capillary effect in millimeters in a glass Tube of 4 mm diameter, when immersed in (i) Water , and (ii) Mercury.
The temperature of the liquid is 20 o C and the values of the surface tension of water and mercury at 20o C contact with air are 0.073575 N/m and 0.51 N/m respectively.

The angle of contact for water is zero and for mercury is 130 degree. Take density of water at 20 o C equal to 998 kg / m3

Solution. Given :

Dia of tube, $d = 4 \text{ mm} = 4 \times 10^{-3}$ m

The capillary effect (*i.e.*, capillary rise or depression) is given by equation (1.20) as

$$h = \frac{4\sigma \cos\theta}{\rho \times g \times d}$$

where σ = surface tension in kgf/m
θ = angle of contact, and ρ = density

(*i*) **Capillary effect for water**

$$\sigma = 0.073575 \text{ N/m}, \theta = 0°$$
$$\rho = 998 \text{ kg/m}^3 \text{ at } 20°\text{C}$$

$$\therefore \quad h = \frac{4 \times 0.073575 \times \cos 0°}{998 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3} \text{ m} = \mathbf{7.51 \text{ mm. Ans.}}$$

(*ii*) **Capillary effect for mercury** -

$$\sigma = 0.51 \text{ N/m}, \theta = 130° \text{ and}$$
$$\rho = \text{sp. gr.} \times 1000 = 13.6 \times 1000 = 13600 \text{ kg/m}^2$$

$$\therefore \quad h = \frac{4 \times 0.51 \times \cos 130°}{13600 \times 9.81 \times 4 \times 10^{-3}} = -2.46 \times 10^{-3} \text{ m} = \mathbf{-2.46 \text{ mm. Ans.}}$$

The negative sign indicates the capillary depression.

Example *The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.*

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

or $$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 = \mathbf{11.045 \text{ N/cm}^2}$. **Ans.**

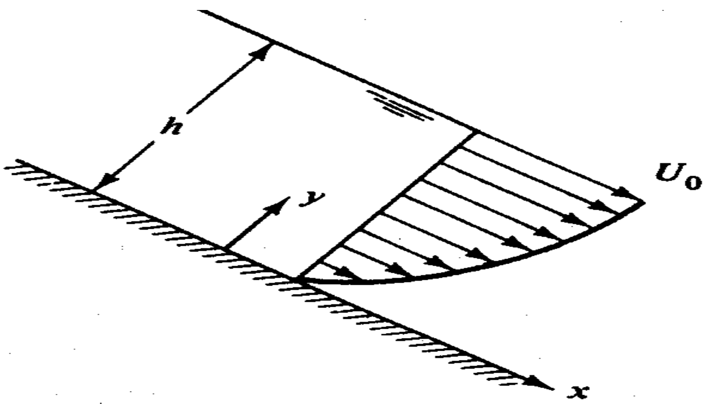
Extra Examples and Problems

Problem (1)

The velocity profile in water flow down a spillway is given by:

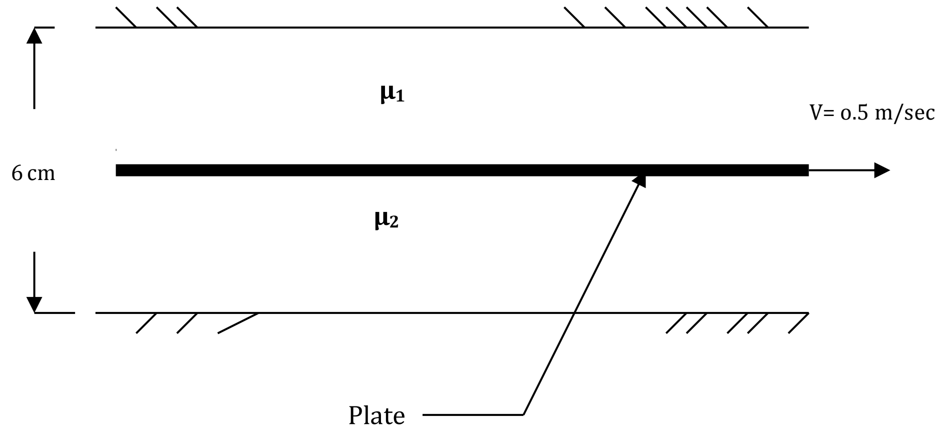
$$u = U_o \left(\frac{h}{y}\right)^{1/7}$$

If $U_o = 1.4$ m/s , h= 3 m and the width is 17 m, how long will it take a volume of water equal to 10^5 m^3 to pass this section of the spillway ?



Problem (2)

A large plate is centrally placed inside a gap which contains two different fluids with ($\mu_1 = 4\ \mu_2$) . If the force per unit area due to shear acting on both side of plate is 50 N . Calculate the values of viscosity for the two fluids (i.e μ_1 and μ_2) .



Example (1) A large dirigible having a volume of 90,000 m^3 contains helium under standard atmospheric conditions [pressure = 101 kPa (abs) and temperature = 15 °C]. Determine the density and total weight of the helium.

volume $= 90{,}000\ m^3$

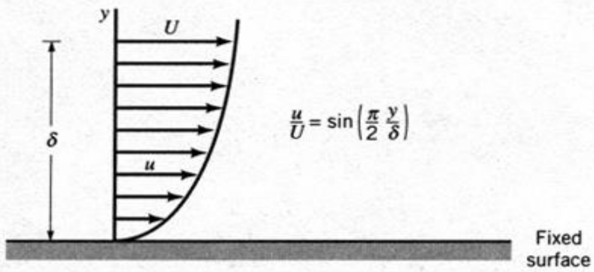
From the ideal gas law,

$$\rho = \frac{p}{RT} = \frac{101\times10^3\ \frac{N}{m^2}}{\left(2077\ \frac{J}{kg\cdot K}\right)\left[(15^\circ C + 273)K\right]} = \underline{\underline{0.169\ \frac{kg}{m^3}}}$$

$$\text{weight} = \rho g \times \text{volume} = \left(0.169\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)\left(9\times10^4\ m^3\right)$$

$$= \underline{\underline{1.49\times10^5\ N}}$$

Example (2) A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of $4 \times 10^{-4}\ m^2/s$ flows past a fixed surface. The velocity profile near the surface is shown in Figure . Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of U and δ, with U and δ expressed in units of meters per second and meters, respectively.



$$\tau_{\substack{surface \\ (y=0)}} = \mu\left(\frac{du}{dy}\right)_{y=0} \quad \text{where } \mu = \nu\rho$$

$$\frac{du}{dy} = \frac{\pi}{2}\frac{U}{\delta}\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

At $y=0$, $\dfrac{du}{dy} = \dfrac{\pi}{2}\dfrac{U}{\delta}(1)$

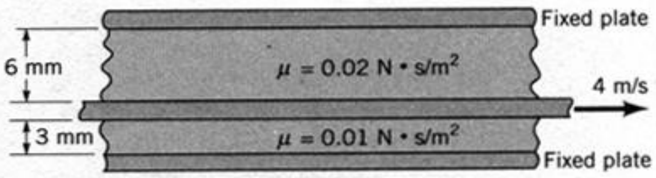
Since, $\mu = \nu\rho$ where $\rho = SG\,\rho_{H_2O} = 0.92\left(1000\ \frac{kg}{m^3}\right)$

$$\tau_{surface} = \nu\rho\left(\frac{\pi}{2}\frac{U}{\delta}\right)$$

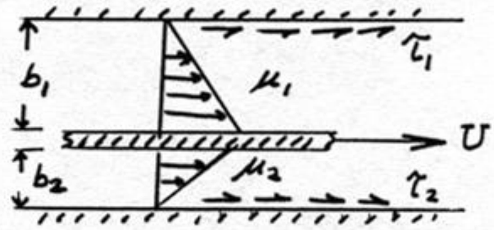
$$= \left(4\times10^{-4}\ \frac{m^2}{s}\right)\left(0.92\times10^3\ \frac{kg}{m^3}\right)\left(\frac{\pi}{2}\right)\frac{U}{\delta}$$

$$= 0.578\ \frac{U}{\delta}\ N/m^2 \quad \text{acting to right on plate}$$

Example (3) A large movable plate is located between two large fixed plates as shown in Fig. Two Newtonian fluids having the viscosities indicated are contained between the plates. Determine the magnitude and direction of the shearing stresses that act on the fixed walls when the moving plate has a velocity of 4 m/s as shown. Assume that the velocity distribution between the plates is linear.



***Solution*:**



$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{b} \text{ so that}$$

$$\tau_1 = \mu_1 \frac{U}{b_1} = \left(0.02 \frac{N \cdot s}{m^2}\right)\left(\frac{4 \frac{m}{s}}{0.006 m}\right)$$

$$= 13.3 \frac{N}{m^2}$$

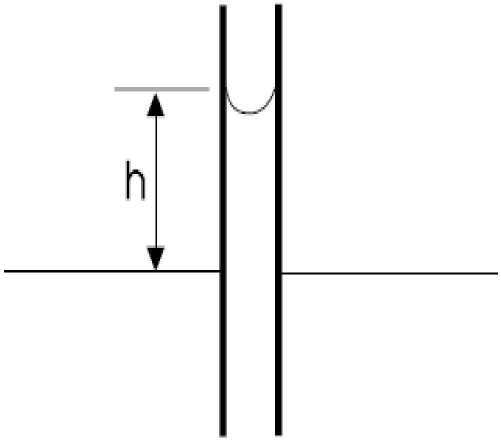
$$\tau_2 = \mu_2 \frac{U}{b_2} = \left(0.01 \frac{N \cdot s}{m^2}\right)\left(\frac{4 \frac{m}{s}}{0.003 m}\right)$$

$$= 13.3 \frac{N}{m^2}$$

Stresses act on fixed walls in direction of moving plate.

Example(4)

Two parallel glass plates separated by 0.5 mm are placed in water at 20°C. The plates are clean, and the width/separation ratio is large so that end effects are negligible. How far will the water rise between the plates?



Solution

The surface tension at 20°C is 7.3×10^{-2} N/m. The weight of the water in the column h is balanced by the surface tension force.

$$whd\rho g = 2w\sigma \cos\theta$$

where w is the width of the plates and d is the separation distance. For water against glass, $\cos\theta \simeq 1$. Solving for h gives

$$h = \frac{\sigma}{d\rho g} = \frac{2 \times 7.3 \times 10^{-2}\ \text{N/m}}{0.5 \times 10^{-3}\ \text{m} \times 998\ \text{kg/m}^3 \times 9.81\ \text{m/s}^2}$$

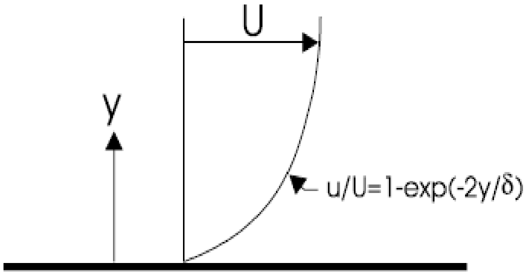
$$= 0.0149\ \text{m} = \underline{\underline{29.8\ \text{mm}}}$$

Example (5)

Air at 15°C forms a boundary layer near a solid wall. The velocity distribution in the boundary layer is given by

$$\frac{u}{U} = 1 - \exp(-2\frac{y}{\delta})$$

where $U = 30$ m/s and $\delta = 1$ cm. Find the shear stress at the wall ($y = 0$).



Solution

The shear stress at the wall is related to the velocity gradient by

$$\tau = \mu \frac{du}{dy} \mid_{y=0}$$

Taking the derivative with respect to y of the velocity distribution

$$\frac{du}{dy} = 2\frac{U}{\delta} \exp(-2\frac{y}{\delta})$$

Evaluating at $y = 0$

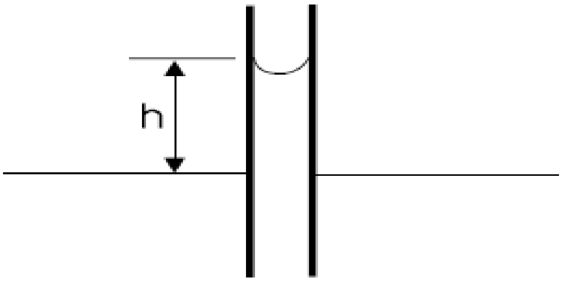
$$\frac{du}{dy} \mid_{y=0} = 2\frac{U}{\delta} = 2 \times \frac{30}{0.01} = 6 \times 10^3 \text{ s}^{-1}$$

From Table A.2, the density of air is 1.22 kg/m^3, and the kinematic viscosity is 1.46×10^{-5} m^2/s. The absolute viscosity is $\mu = \rho\nu = 1.22 \times 1.46 \times 10^{-5}$ $= 1.78 \times 10^{-5}$ N·s/m^2. The shear stress at the wall is

$$\tau = \mu \frac{du}{dy} \mid_{y=0} = 1.78 \times 10^{-5} \times 6 \times 10^3 = \underline{\underline{0.107 \text{ N/m}^2}}$$

Example (1)

Two parallel glass plates separated by 0.5 mm are placed in water at 20°C. The plates are clean, and the width/separation ratio is large so that end effects are negligible. How far will the water rise between the plates?



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The surface tension at 20°C is 7.3×10^{-2} N/m. The weight of the water in the column h is balanced by the surface tension force.

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where w is the width of the plates and d is the separation distance. For water against glass, $\cos\theta \simeq 1$. Solving for h gives

$$h = \frac{\sigma}{d\rho g} = \frac{2 \times 7.3 \times 10^{-2} \text{ N/m}}{0.5 \times 10^{-3} \text{ m} \times 998 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2}$$
$$= 0.0149 \text{ m} = \underline{\underline{29.8 \text{ mm}}}$$

Fluid Static

Hydrostatic Forces

Case 1. Forces from a uniform pressure distribution

- **Vertical Force**

Force = weight of fluid, W

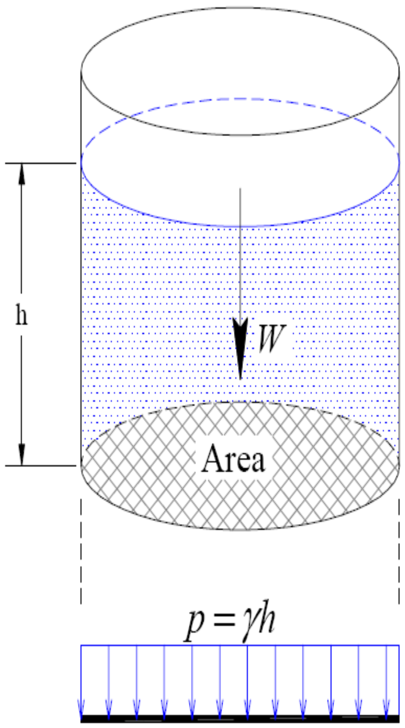
$W = \gamma \forall = \gamma(A \times h)$

Pressure = Force / area

$$p = \frac{W}{A} = \frac{\gamma(A \times h)}{A}$$

$p = \gamma h$

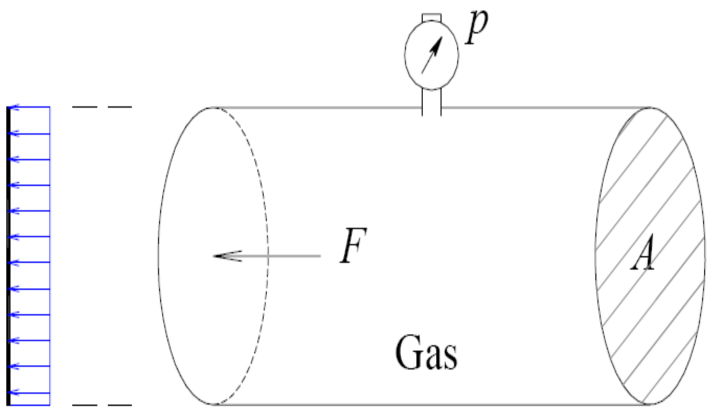
The pressure distribution on the surface with area (A) is uniform (the pressure is equal at each and every point on the surface).



- **Horizontal Force**

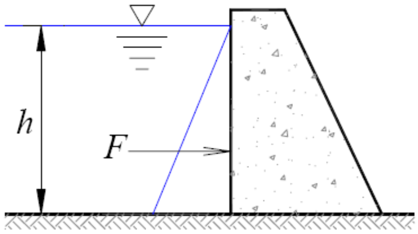
The pressure force resulted from a gas tank on the vertical walls is horizontal and uniform.

$F = pA$



Case 2. Forces from a non-uniform pressure distribution.

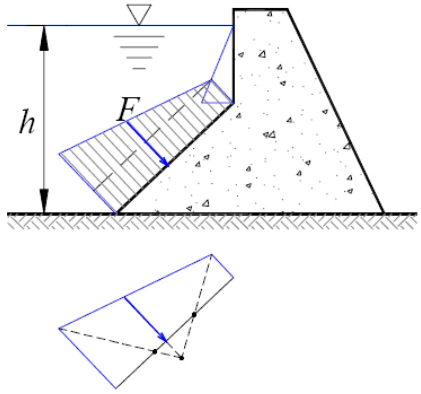
Horizontal Force



Pressure varies with depth. It starts at zero at the surface, and reaches maximum at the base.

Triangular distribution of water pressure on a vertical wall.

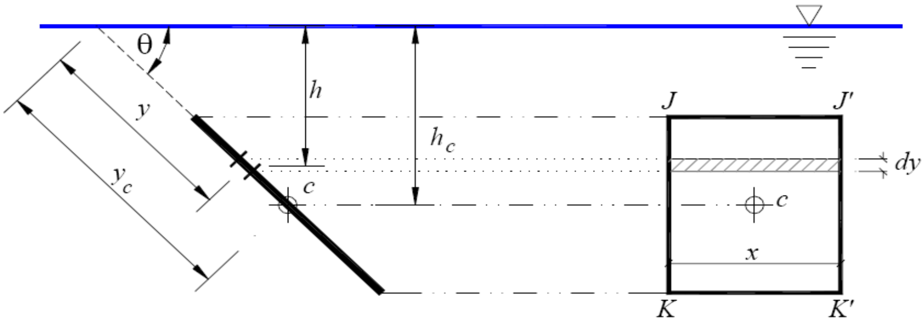
Angled Force



Trapezoidal pressure distribution on an inclined surface.

Hydrostatic Forces on Plane Surfaces

Step 1. Find the resultant force.



Choose an element of area so that the pressure on it is uniform. Such an element is a horizontal strip with a width equal to x, so, $dA = xdy$. Notice that the width of the surface is not constant.

Total force on the whole surface will equal to the summation of forces on all elements of dy:

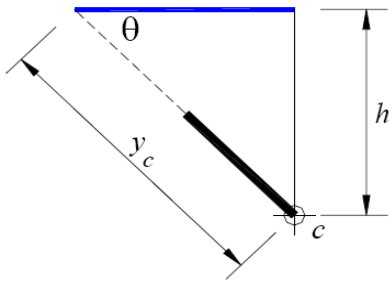
Total force = F = $\sum dF$

Force = pressure × area

$$F = \sum pdA$$

The mathematical equivalent to the summations is integration (when the strip height "dy" is too small)

$$F = \int pdA \qquad p = \gamma h$$



$$h = y\sin\theta \Rightarrow p = \gamma(y\sin\theta)$$

$$\Rightarrow F = \int(\gamma \times y\sin\theta)dA \qquad F = \gamma\sin\theta\int ydA$$

$\int ydA$ is the mathematical expression for the ***first moment of area***. The first moment of area equals to the sum of the products of area times distance to the centroid.

$\int y dA = y_c A$

Where:

y_c the distance from the fluid surface to the centroid of the plan surface along the inclined plan surface (in vertical surfaces, $y_c = h_c$).

➔ $F = \gamma \sin\theta (y_c A)$, but $h = y \sin\theta$

$$\boxed{F = \gamma (h_c A) \qquad \ldots\ldots\ldots \#}$$

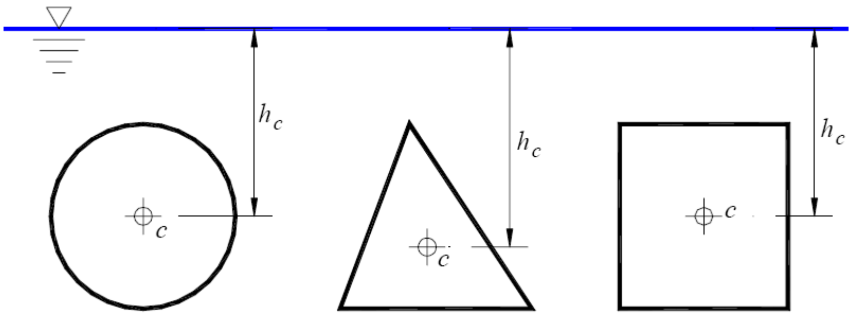
Where:

F Pressure force (normal to the surface)

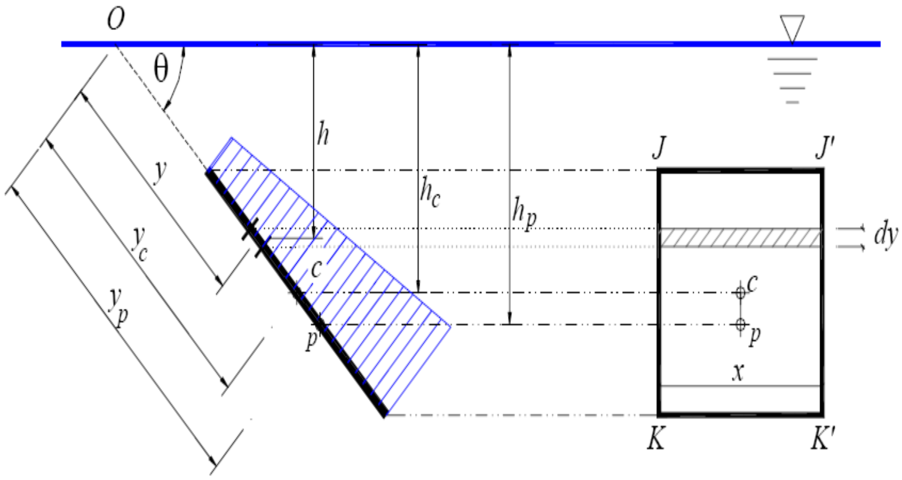
γ ***Specific weight of fluid (for water, γ = 9.81 kN/m3 "SI Units", and γ = 62.4 lb/ft3 "BG Units")***

A Cross sectional area of the plan surface.

h_c <u>Vertical</u> distance from the surface of fluid to the centroid of the plan surface.



Step 2. Find the location where the resultant force is applied.



Force = pressure × area

$dF = pdA \qquad p = \gamma h \qquad h = y\sin\theta$ ➔ $p = \gamma(y\sin\theta)$

$dF = \gamma(y\sin\theta)dA$

To find the point of application of the pressure force, take the summation of moments of forces at O:

$\sum M_o = \sum y.dF$

Substitute for the value of dF:

$\sum M_o = \sum y[\gamma(y\sin\theta)]dA$

The summation can be replaced with integration when the step is too small:

$\sum M_o = \int \gamma(y^2 \sin\theta)dA$

For the whole system to be stable, the sum of moments should equal to the moment of the pressure force F ($F \times y_p$)

Where y_p is the distance from the fluid surface to the center of pressure of the surface along the inclined plan.

$F \times y_p = \int \gamma(y^2 \sin\theta)dA$ ➔ $F \times y_p = \gamma \sin\theta \int y^2 dA$

$\int y^2 dA$ is the second moment of area (moment of inertia of the area) about $\boldsymbol{O} = I_o$

And $F = \gamma h_c A = \gamma(y_c \sin\theta)A$

$(\gamma \times y_c \sin\theta \times A)y_p = \gamma \sin\theta \times I_o$

$$y_p = \frac{I_o}{y_c A} \qquad \ldots\ldots\ldots \#$$

The above equation can be expressed in another form:

$I_o = A y_c^{\ 2} + I_c$ (***Parallel axes theorem***)

If a body has moment of inertia $\boldsymbol{I_c}$ about an axis $\boldsymbol{C}$ through its center of mass (C. G.), and if $\boldsymbol{O}$ is another axis parallel to $\boldsymbol{C}$ at a distance $\boldsymbol{y_c}$ from it, then the body's moment of inertia about $\boldsymbol{O}$ is $I_o = A y_c^{\ 2} + I_c$.

Then:

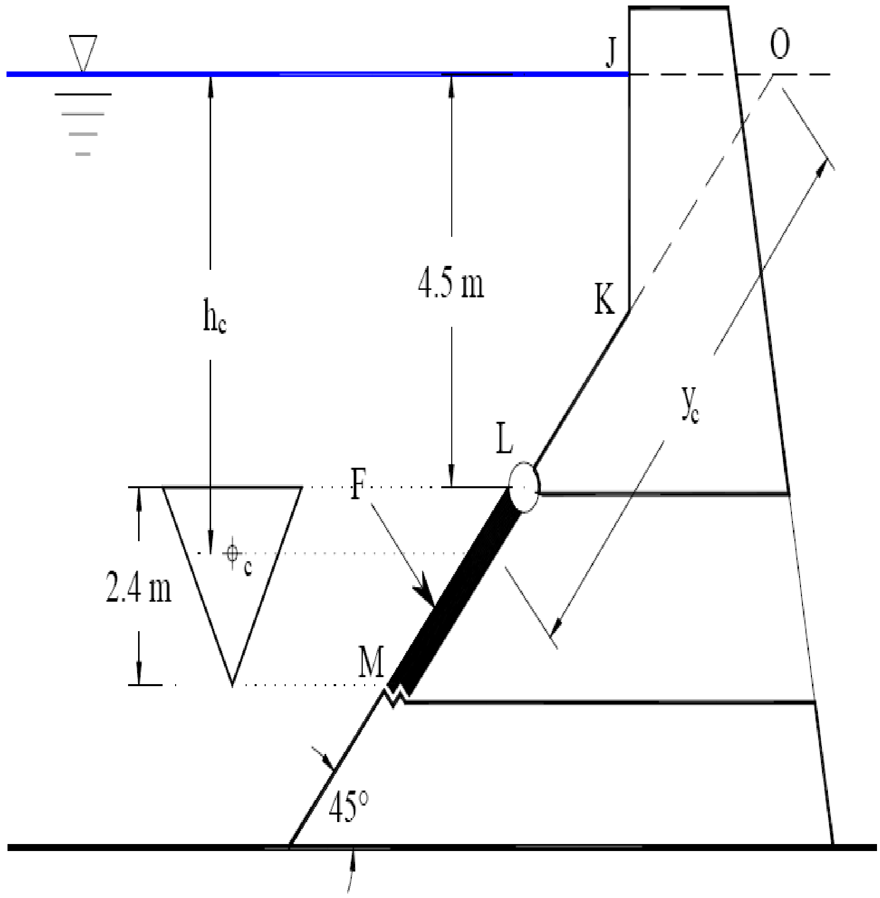
$$y_p = \frac{A y_c^{\ 2} + I_c}{y_c A}$$

$$y_p = y_c + \frac{I_c}{y_c A}$$

Summary of Results

- Hydrostatic Force: $F = \gamma y_c (\sin\theta) A = \gamma h_c A$
- Acting at Location: $y_p = y_c + \dfrac{I_c}{y_c A}$

Example. The small dam shown in the figure is 8.5 m wide. An equilateral triangular gate (LM) that is 2.4-m in height is located along the lower face (KN) so that the apex is at M. Depth of water above the hinge (L) is 4.5 m. Calculate the magnitude and point of application of the force acting on side LM.

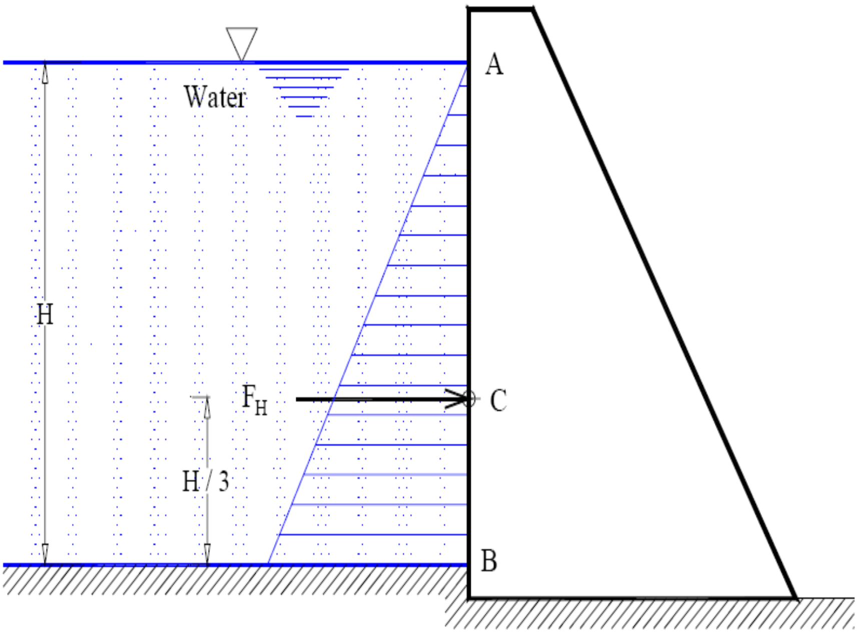


Supplementary Notes

Hydrostatic forces

1- Forces on vertical plan surfaces:

a. The surface intersects with fluid surface



- **Force magnitude:**

$$F_H = \gamma . h_c . A$$

Where:

γ Specific weight of fluid (for water, γ = 9.81 kN/m3 "SI Units", and γ = 62.4 lb/ft3 "BG Units")

hc Vertical distance from the surface of fluid to the centroid of the plan surface.

A cross sectional area of the plan surface (= distance AB × width).

$$F_H = \gamma\left(\frac{H}{2}\right)(H \times W)$$

Where

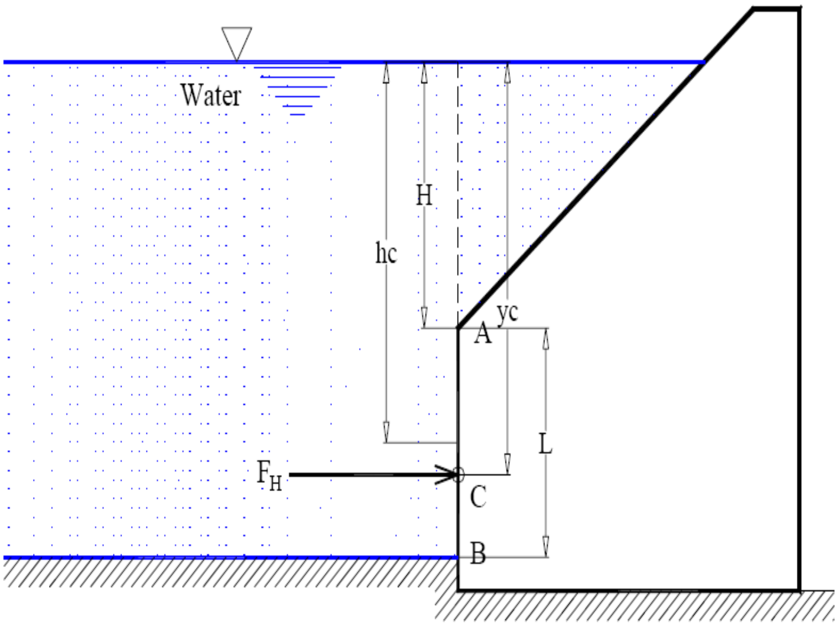
W width of the plan surface (and you can take it equal to unity, and that will give results per unit width)

$$F_H = \frac{1}{2}\gamma.H^2$$

- **Point of action:**

The line of action of the force is at distance H/3 from the bottom.

b. The plan surface does not intersect with the fluid surface:



- **Force magnitude:**

$$F_H = \gamma.h_c.A$$

$$h_c = H + \frac{L}{2}$$

- **Point of action:**

The line of action of the force is at distance = y_c, from the fluid surface:

$$y_p = y_c + \frac{I_c}{y_c.A}$$

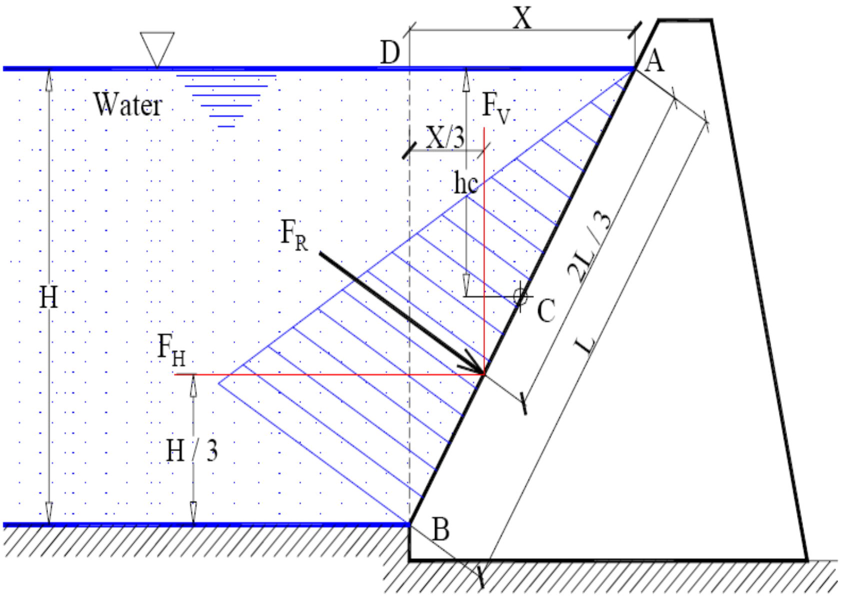
Where:

y_p The distance from the fluid surface to the center of pressure of the surface along the inclined plan.

y_c the distance from the fluid surface to the centroid of the plan surface along the inclined plan surface (in vertical surfaces, $y_c = h_c$).

I_c Moment of inertia of the plan surface along its centroidal axis.

2- Forces on inclined plan surfaces



The pressure force is always perpendicular to the surface, and for easy dealing with that type of problems, the resultant force is resolved into two components (Horizontal, and vertical).

$$F_H = \gamma.h_c.A$$

Where:

A is the vertical projection of the plan surface = H * width

And the line of action of (F_H) is at a distance = H/3 from the bottom.

The vertical force (F_v) is equal to the weight of fluid over the inclined surface AB (= area **ABD** × width × γ)

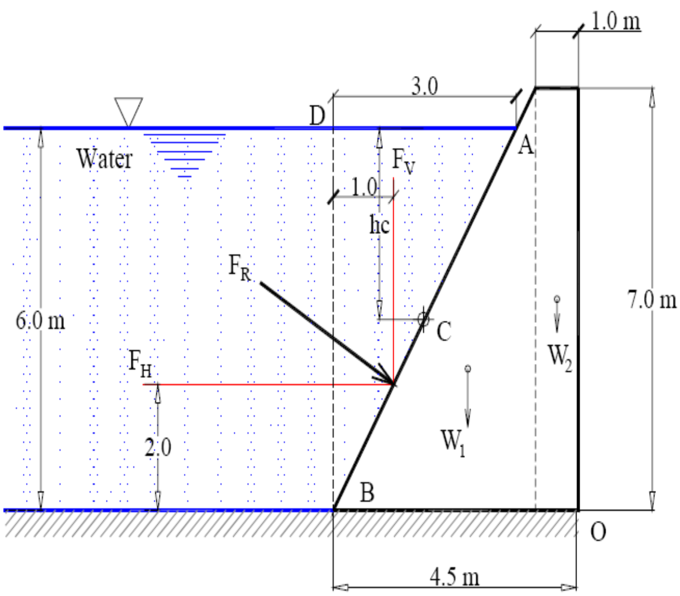
The line of action of F_v passes through the same point of action of F_H, at a distance = X/3 to the right of line BD.

Application:

Dam Stability:

Example:

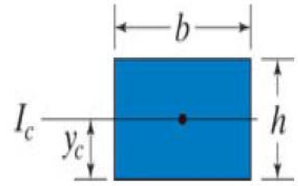
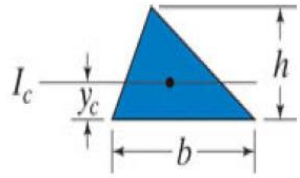
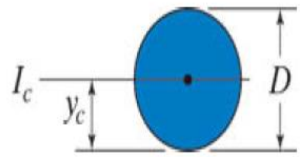
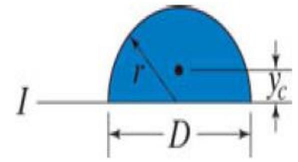
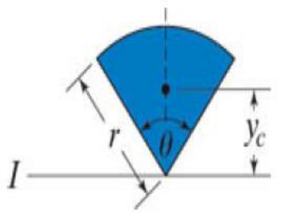
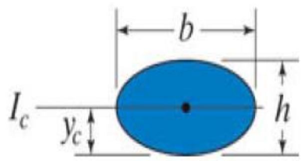
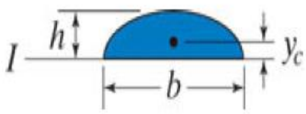
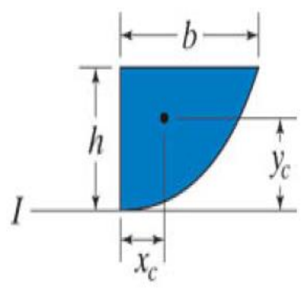
A concrete dam retaining 6.0 m of water shown in the figure. The unit weight of the concrete is 23.50 kN/m³. The foundation soil is impermeable. Determine:
a- The factor of safety against sliding
b- The factor of safety against overturning.
The coefficient of friction between the base of the dam and the foundation soil is 0.48.



The forces acting on the dam are:

1- Horizontal water pressure (F_H),
2- Vertical water pressure (F_V),
3- Weight of the dam material = $W_1 + W_2$

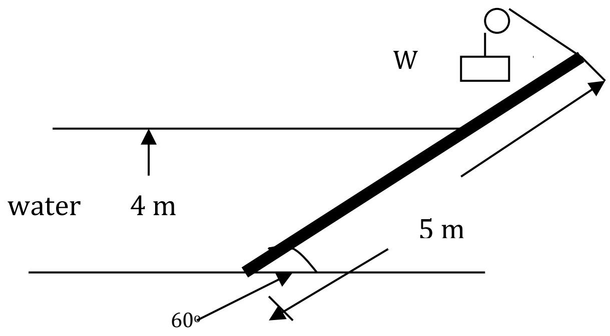
Properties of areas

Shape	Sketch	Area	Location of Centroid	Ic
		$b\times h$	$y_c=\frac{h}{2}$	$I_c=\frac{bh^3}{12}$
		$\frac{bh}{2}$	$y_c=\frac{h}{3}$	$I_c=\frac{bh^3}{36}$
		$\frac{\pi D^2}{4}$	$y_c=\frac{D}{2}$	$I_c=\frac{\pi D^4}{64}$
		$\frac{\pi D^2}{8}$	$y_c=\frac{4r}{3\pi}$	$I=\frac{\pi D^4}{128}$
		$\frac{\theta r^2}{2}$	$y_c=\frac{4r}{3\theta}\sin\frac{\theta}{2}$	$I=\frac{r^4}{8}(\theta+\sin\theta)$
		$\frac{\pi bh}{4}$	$y_c=\frac{h}{2}$	$I_c=\frac{\pi bh^3}{64}$
		$\frac{\pi bh}{4}$	$y_c=\frac{4h}{3\pi}$	$I=\frac{\pi bh^3}{16}$
		$\frac{2bh}{3}$	$x_c=\frac{3b}{8}$ $y_c=\frac{h}{2}$	$I=\frac{2bh^3}{7}$

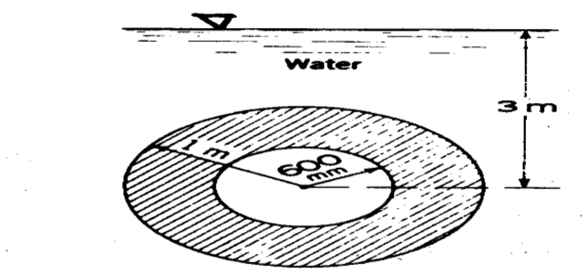
Problems:

Problem (1)

The gate shown is 3.0 wide. Find the value of the weight required for the gate to be in equilibrium.

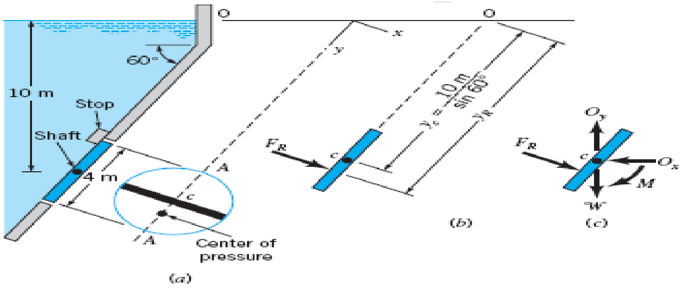


Problem (2) Find the force exerted by water on one side of the vertical annular disk shown below. Also calculate the center of pressure.



Example (1)

The 4-m-diameter circular gate of Fig. is located in the inclined wall of a large reservoir containing water ($\gamma = 9.80$ kN/m^3). The gate is mounted on a shaft along its horizontal diameter. For a water depth of 10 m above the shaft determine: (a) the magnitude and location of the resultant force exerted on the gate by the water, and (b) the moment that would have to be applied to the shaft to open the gate.



(a) To find the magnitude of the force of the water we can apply

$$F_R = \gamma h_c A$$

and since the vertical distance from the fluid surface to the centroid of the area is 10 m it follows that

$$F_R = (9.80 \times 10^3 \text{ N/m}^3)(10 \text{ m})(4\pi \text{ m}^2)$$
$$= 1230 \times 10^3 \text{ N} = 1.23 \text{ MN}$$

To locate the point (center of pressure) through which F_R acts,

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad y_R = \frac{I_{xc}}{y_c A} + y_c$$

To obtain y_R, we have

$$I_{xc} = \frac{\pi R^4}{4}$$

and y_c is shown in Fig. Thus,

$$y_R = \frac{(\pi/4)(2 \text{ m})^4}{(10 \text{ m}/\sin 60°)(4\pi \text{ m}^2)} + \frac{10 \text{ m}}{\sin 60°}$$
$$= 0.0866 \text{ m} + 11.55 \text{ m} = 11.6 \text{ m}$$

and the distance (along the gate) below the shaft to the center of pressure is

$$y_R - y_c = 0.0866 \text{ m}$$

We can conclude from this analysis that the force on the gate due to the water has a magnitude of 1.23 MN and acts through a point along its diameter A-A at a distance of 0.0866 m (along the gate) below the shaft. The force is perpendicular to the gate surface as shown.

(b) The moment required to open the gate can be obtained with the aid of the free-body diagram . In this diagram $\mathcal{W}$ is the weight of the gate and O_x and O_y are the horizontal and vertical reactions of the shaft on the gate. We can now sum moments about the shaft

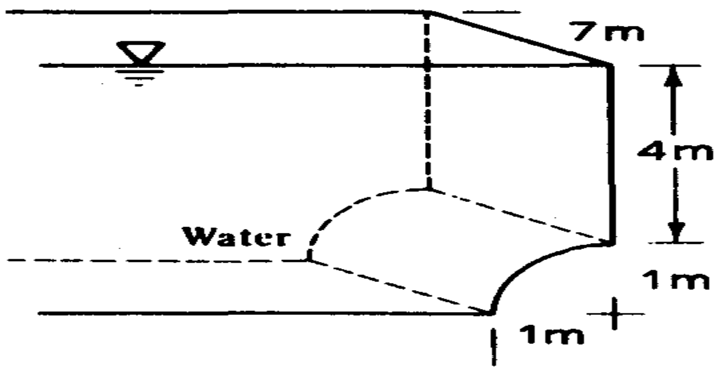
$$\sum M_c = 0$$

and, therefore,

$$M = F_R\,(y_R - y_c)$$
$$= (1230 \times 10^3 \text{ N})(0.0866 \text{ m})$$
$$= 1.07 \times 10^5 \text{ N} \cdot \text{m}$$

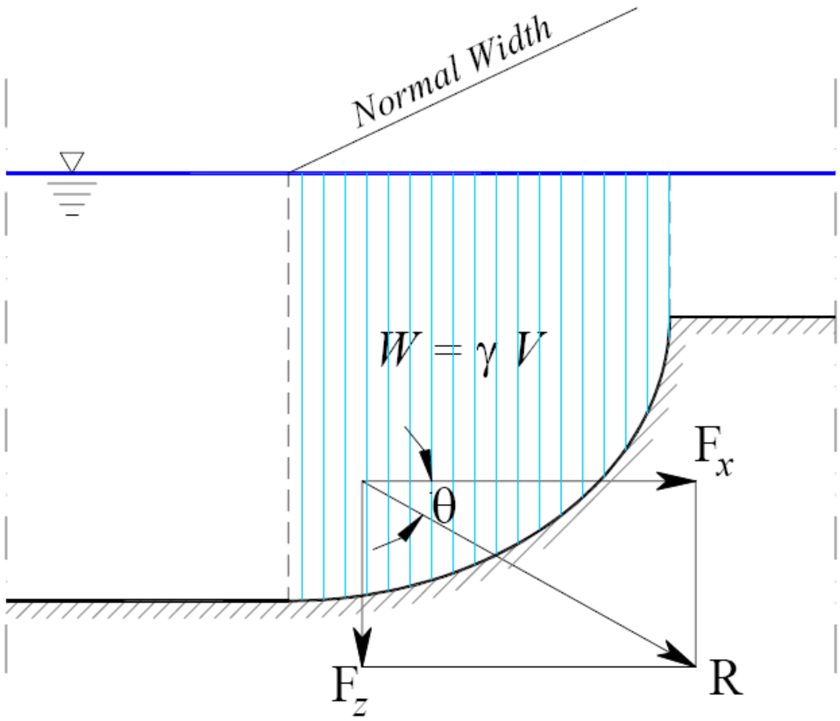
Problem (3)

Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle face of the tank shown below.



Hydrostatic Force on a Curved Submerged Surface

Because of the curvature of the surface, the direction of the resultant force is not pre-determined as in the previous case. One needs to decompose the force to its horizontal and vertical components (F_x, and F_z) and find the points of application for each.



The horizontal component, F_x (of the resultant force on a curved submerged surface) is found by taking the projection of the curved surface onto a vertical place and finding the force and its point of application based on the formulation above.

$$F_x = \gamma h_c A$$

Where A is the vertical projection area of the curved surface.

The vertical component, F_z (of the resultant force on a curved submerged surface) is the weight of the fluid <u>above</u> the curved surface up to the free surface.

$$F_z = W = \gamma \forall$$

Where V is the volume of water over the curved surface up to the water surface.

Resultant force

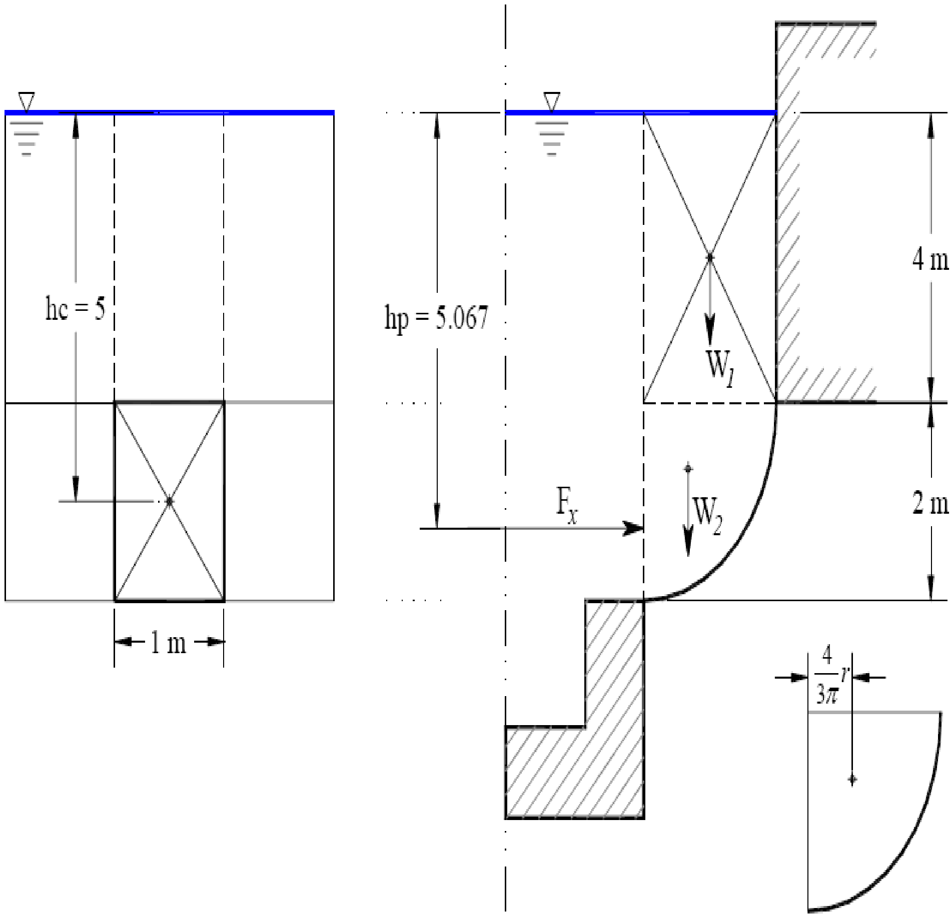
$$R = \sqrt{F_x^{\,2} + F_z^{\,2}}$$

The angle the resultant force makes to the horizontal is

$$\theta = \tan^{-1}\left(\frac{F_z}{F_x}\right)$$

Example 1: ***Hydrostatic force on a curved surface.***
Given: R = 2.0 m and h = 4.0 m Find: The resultant force on the curved surface per unit length.

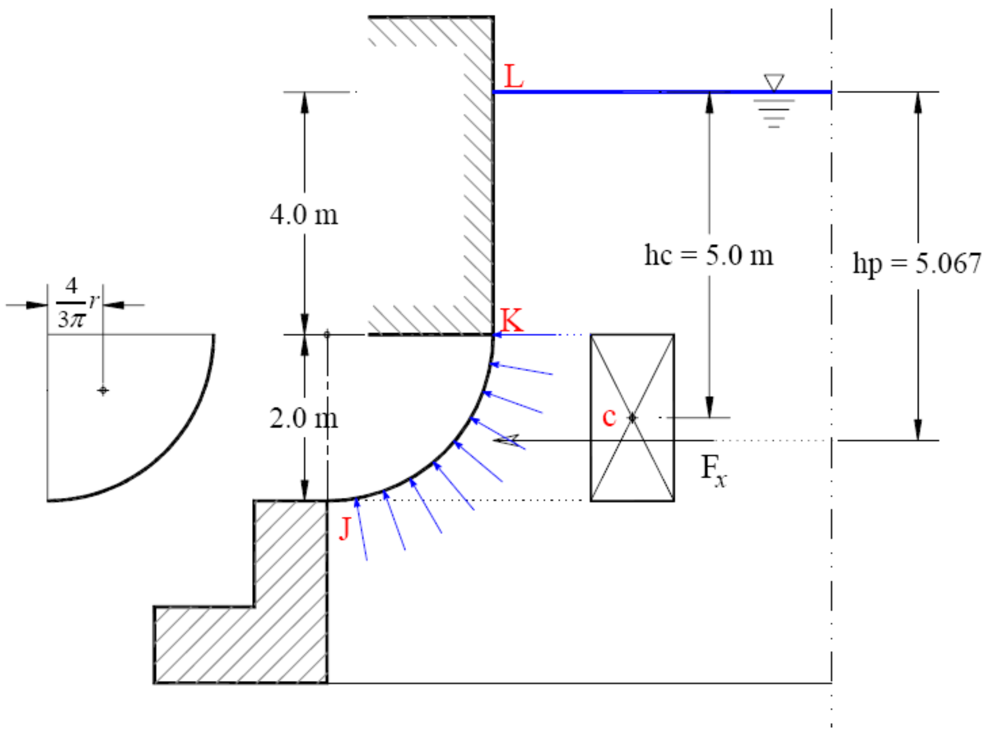
Solution:



***Example 2:* Hydrostatic force on a curved surface.**
Given: R = 2.0 m and h = 4.0 m . Find: The resultant force on the curved surface per unit length

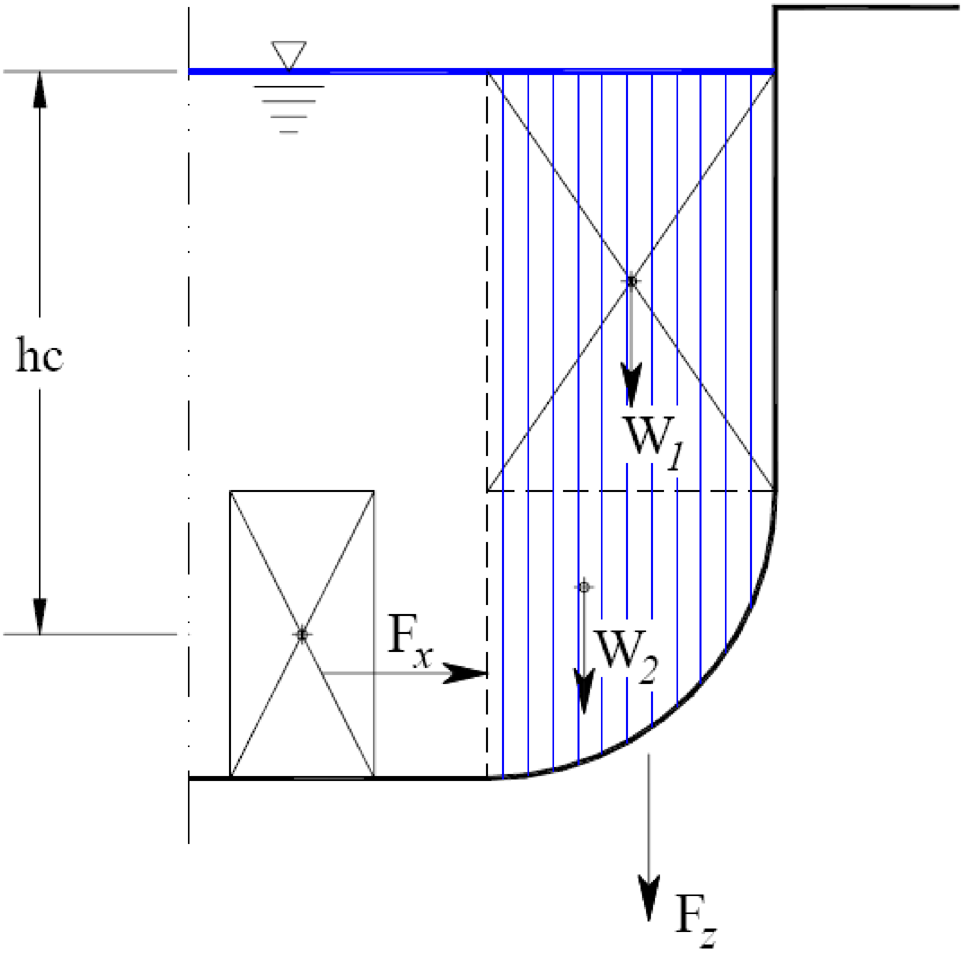
Solution:

Note that water is on the right side this time



Summary of pressure on curved surfaces

Case 1: Liquid overtop the curved surface:



1- Horizontal component of the hydrostatic force:
Treat the problem as if it was a plain, vertical surface
Magnitude:

$$F_x = \gamma h_c A$$

Acting at location:

$$y_p = y_c + \frac{I_c}{y_c A}$$

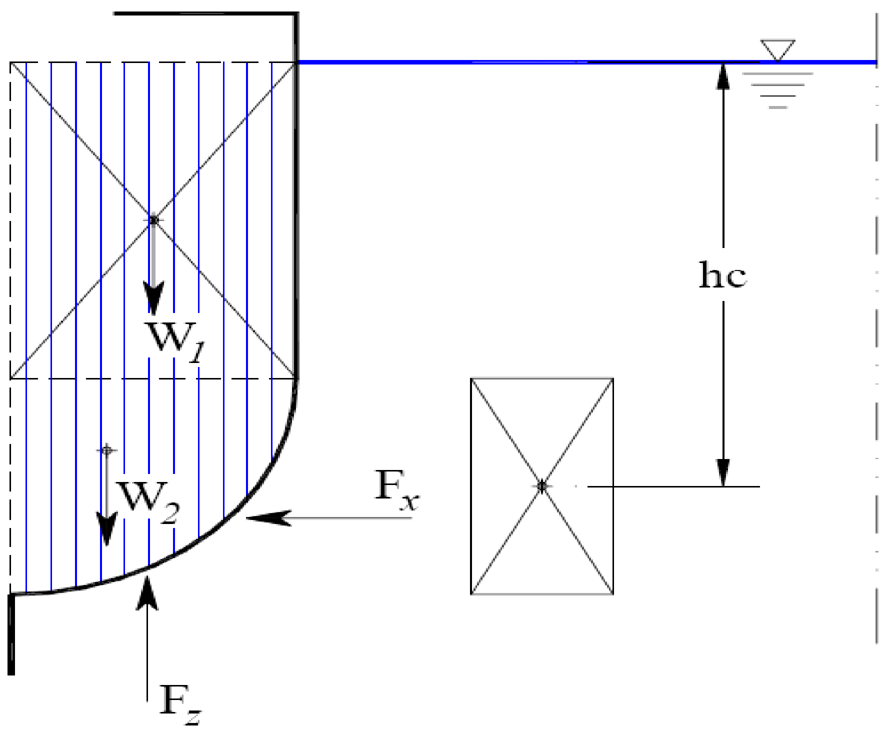
2- Vertical component of the hydrostatic force

Calculate weight of the block of fluid above the surface
Magnitude

$$F_z = W = \gamma \forall$$

Acting at location x_p (distance of the resultant of water weight to a specific datum)

Case 2: Liquid underneath the curved surface:



1- Horizontal component of the hydrostatic force:

Treat the problem as if it was a plain, vertical surface
Magnitude:

$$F_x = \gamma h_c A$$

Acting at location:

$$y_p = y_c + \frac{I_c}{y_c A}$$

2- Vertical component of the hydrostatic force
Calculate the weight of the block of fluid displaced by the surface
Magnitude

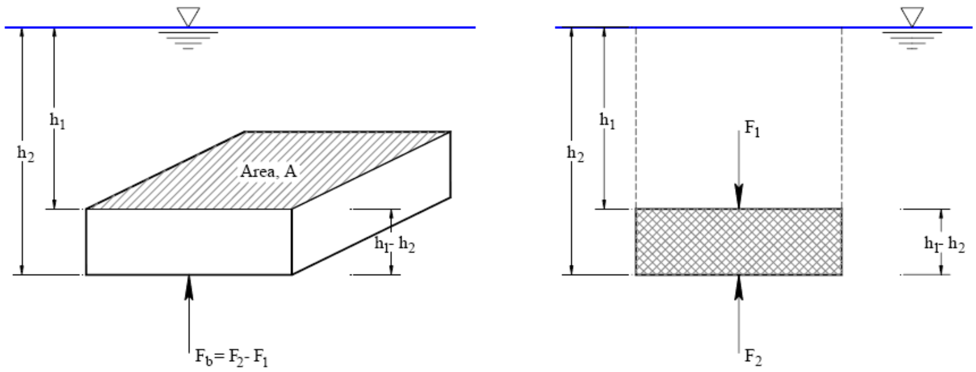
$$F_z = W = \gamma \forall$$

Acting at x_p location (distance of the resultant of water weight to a specific datum)

Buoyancy

F_B = buoyant force = net vertical force exerted by fluid on an immersed or floating body

Case 1: Submerged object



The water pressure is equally distributed all over the submerged body, but because the direction of moving force (gravity) is downward, we take the balance of the acting forces in the vertical direction.
The water pressure force acting on the top surface (F_1) is equal to the weight of the volume of fluid above the top surface.

$$F_1 = \gamma \times V_1 = \gamma A h_1$$

F_2 is equal to the weight of the volume of fluid above the bottom surface.

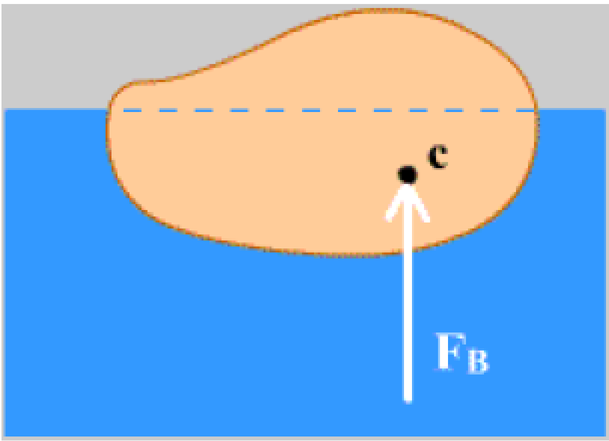
$$F_2 = \gamma \times \cancel{V}_2 = \gamma A h_2$$

The difference between the two forces is equal to the buoyancy force Fb.

$$F_B = F_2 - F_1 = \gamma(\cancel{V}_2 - \cancel{V}_1)$$

The difference between the volumes is equal to the volume of the submerged body, which is equal to the volume of the displaced fluid.

The basic principle of buoyancy is **Archimedes**' Principle: The buoyant force acting on a submerged body is equal to the weight of the fluid displaced by the body.



$$F_B = \gamma \times \cancel{V}$$

Where:

F_B	Buoyant force	lb, or N
γ	Unit weight of the displaced liquid	$\frac{lb}{ft^3}$, or N/m^3
$\forall$	Volume of the displaced liquid	ft^3, or m^3.

Archimedes made use of the principle to find out if the crown of the king was made of pure gold, or made of gold and silver. Can you tell how??
He weighted the crown (W_1), and then got an equivalent weight of pure gold. Assuming that the density of both the crown, and the pure gold are the same, the volume of them should be the same:

$$\rho = \frac{W}{\forall} \qquad\qquad \forall = \frac{W}{\rho}$$

So, he placed them in water, and watched for the displaced volume. If the two objects displaced the same volume of water, then they are made of the same material.

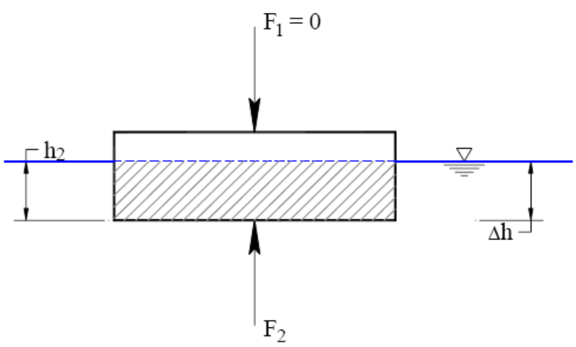
Case 2: Floating object

$$F_B = F_2 - F_1 = \gamma(\forall_2 - \forall_1)$$

F1 = 0

$$F_B = F_2 = \gamma \Delta h \times A$$

$$F_B = \gamma \times \forall$$



Where $\forall$ is the submerged volume = volume of displaced liquid.

If W is the weight of the body, then:

If	$W = F_B$	➔	Object floats
	$W > F_B$	➔	Object sinks
	$W < F_B$	➔	Object rises

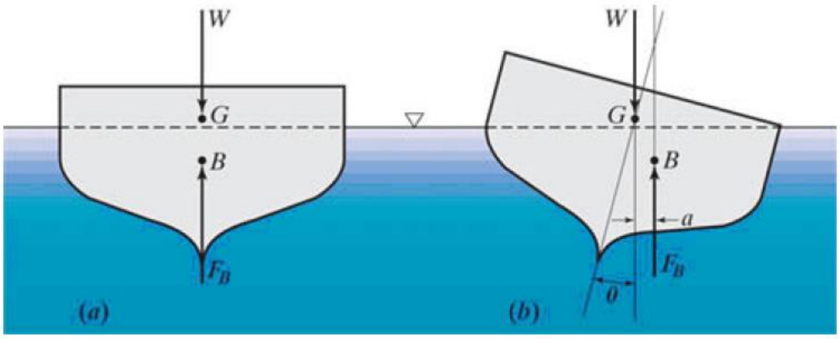
Direction and Location of Forces

Force	**Point of action**
Weight Buoyancy, FB	Center of gravity of the body Center of buoyancy

Stability of floating objects

There are two forces involved in floating body stability:

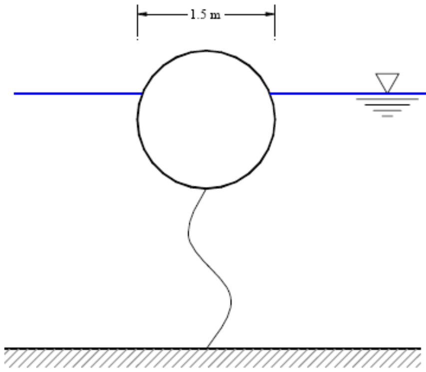
Weight, and Buoyancy

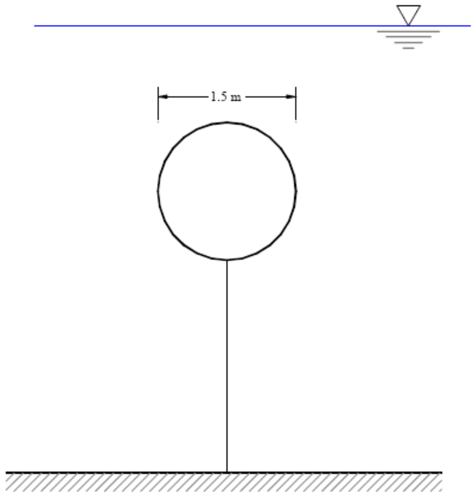


If the moment produced by these two forces is a righting moment, the floating body will be stable.

Example Problem: Buoyancy Force

Given: A buoy is attached to the floor of a bay with a flexible cable. The buoy is designed to float
in seawater (γ = 10.1 kN/m3) during both high and low tide. During a storm surge, however, the spherical buoy may become completely submerged.
Find: The tension in the cable, if the 1.50 m diameter buoy weighs 8.5kN becomes submerged.





Example 5

2.88 Circular-arc *Tainter* gate ABC pivots about point O. For the position shown, determine (a) the hydrostatic force on the gate (per meter of width into the paper); and (b) its line of action. Does the force pass through point O?

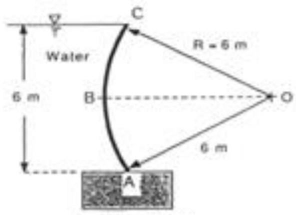
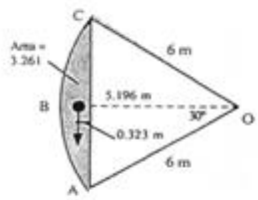


Fig. P2.88

Solution: The horizontal hydrostatic force is based on vertical projection:

$$F_H = \gamma h_{CG} A_{vert} = (9790)(3)(6\times 1) = 176220 \text{ N} \quad \text{at 4 m below C}$$

The vertical force is *upward* and equal to the weight of the missing water in the segment ABC shown shaded below. Reference to a good handbook will give you the geometric properties of a circular segment, and you may compute that the segment area is 3.261 m^2 and its centroid is 5.5196 m from point O, or 0.3235 m from vertical line AC, as shown in the figure. The vertical (upward) hydrostatic force on gate ABC is thus



$$F_V = \gamma A_{ABC}(\text{unit width}) = (9790)(3.2611)$$
$$= 31926 \text{ N} \quad \text{at 0.4804 m from B}$$

The net force is thus $F = [F_H^2 + F_V^2]^{1/2} =$ **179100 N** per meter of width, acting upward to the right at an angle of **10.27°** and passing through a point 1.0 m below and 0.4804 m to the right of point B. This force passes, as expected, *right through point O*.

Example 6

2.89 The tank in the figure contains benzene and is pressurized to 200 kPa (gage) in the air gap. Determine the vertical hydrostatic force on circular-arc section AB and its line of action.

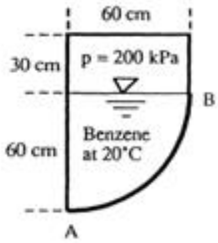


Fig. P2.89

Solution: Assume unit depth into the paper. The vertical force is the weight of benzene plus the force due to the air pressure:

$$F_V = \frac{\pi}{4}(0.6)^2(1.0)(881)(9.81) + (200,000)(0.6)(1.0) = \mathbf{122400\,\frac{N}{m}} \quad \textit{Ans.}$$

Most of this (120,000 N/m) is due to the air pressure, whose line of action is in the middle of the horizontal line through B. The vertical benzene force is 2400 N/m and has a line of action (see Fig. 2.13 of the text) at $4R/(3\pi) = 25.5$ cm to the right or A.

The moment of these two forces about A must equal to moment of the combined (122,400 N/m) force times a distance X to the right of A:

$$(120000)(30\text{ cm}) + (2400)(25.5\text{ cm}) = 122400(X), \quad \textit{solve for}\ \ \mathbf{X = 29.9\,cm} \quad \textit{Ans.}$$

The vertical force is **122400 N/m** (down), acting at **29.9 cm** to the right of A.

Example 7

2.91 The hemispherical dome in Fig. P2.91 weighs 30 kN and is filled with water and attached to the floor by six equally-spaced bolts. What is the force in each bolt required to hold the dome down?

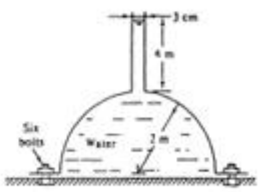


Fig. P2.91

Solution: Assuming no leakage, the hydrostatic force required equals the *weight of missing water*, that is, the water in a 4-m-diameter cylinder, 6 m high, minus the hemisphere and the small pipe:

$$\begin{aligned} F_{total} &= W_{2\text{-m-cylinder}} - W_{2\text{-m-hemisphere}} - W_{3\text{-cm-pipe}} \\ &= (9790)\pi(2)^2(6) - (9790)(2\pi/3)(2)^3 - (9790)(\pi/4)(0.03)^2(4) \\ &= 738149 - 164033 - 28 = 574088\text{ N} \end{aligned}$$

The dome material helps with 30 kN of weight, thus the bolts must supply 574088−30000 or 544088 N. The force in each of 6 bolts is 544088/6 or $F_{bolt} \approx$ **90700 N** *Ans.*

Example 8

2.96 Curved panel BC is a 60° arc, perpendicular to the bottom at C. If the panel is 4 m wide into the paper, estimate the resultant hydrostatic force of the water on the panel.

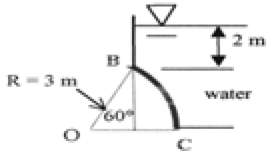
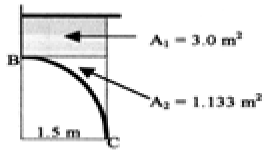


Fig. P2.96

Solution: The horizontal force is,

$$F_H = \gamma h_{CG} A_h$$
$$= (9790\ \text{N/m}^3)[2 + 0.5(3\sin 60°)\ \text{m}]$$
$$\times[(3\sin 60°)\text{m}(4\ \text{m})]$$
$$= 335{,}650\ \text{N}$$



The vertical component equals the weight of water above the gate, which is the sum of the rectangular piece above BC, and the curvy triangular piece of water just above arc BC—see figure at right. (The curvy-triangle calculation is messy and is not shown here.)

$$F_V = \gamma(\text{Vol})_{\text{above BC}} = (9790\ \text{N/m}^3)[(3.0 + 1.133\ \text{m}^2)(4\ \text{m})] = 161{,}860\ \text{N}$$

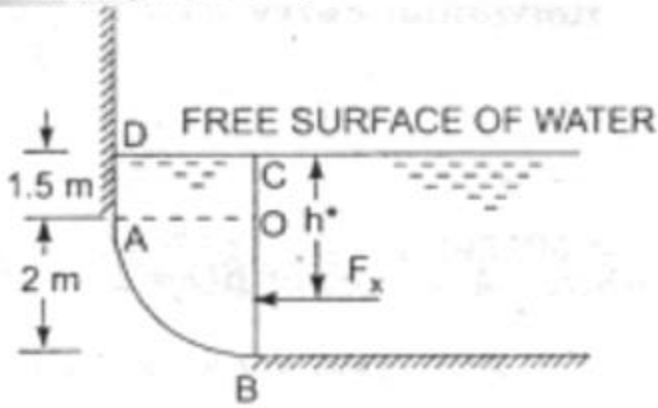
The resultant force is thus,

$$F_R = [(335{,}650)^2 + (161{,}860)^2]^{1/2} = 372{,}635\ \text{N} = \mathbf{373\ kN} \quad \textit{Ans.}$$

This resultant force acts along a line which passes through point O at

$$\theta = \tan^{-1}(161{,}860/335{,}650) = \mathbf{25.7°}$$

Problem 3.22 *Compute the horizontal and vertical components of the total force acting on a curved surface AB, which is in the form of a quadrant of a circle of radius 2 m as shown in Fig. 3.29. Take the width of the gate as unity.*



Solution. Given :

Width of gate $= 1.0$ m

Radius of the gate $= 2.0$ m

$\therefore$ Distance $AO = OB = 2$ m

Horizontal force, F_x exerted by water on gate is given by equation (3.17) as

$F_x =$ Total pressure force on the projected area of curved surface AB on vertical plane

$= $ Total pressure force on OB

{projected area of curved surface on vertical plane $= OB \times 1$}

$$= \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 2 \times 1 \times \left(1.5 + \frac{2}{2}\right) \quad \{\because \text{ Area of } OB = A = BO \times 1 = 2 \times 1 = 2.$$

$\bar{h}$ = Depth of C.G. of OB from free surface $= 1.5 + \frac{2}{2}$ }

$F_x = 9.81 \times 2000 \times 2.5 =$ **49050 N. Ans.**

The point of application of F_x is given by $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

where I_G = M.O.I. of OB about its C.G. $= \frac{bd^3}{12} = \frac{1 \times 2^3}{12} = \frac{2}{3}$ m^4

$$\therefore \quad h^* = \frac{\frac{2}{3}}{2 \times 2.5} + 2.5 = \frac{1}{7.5} + 2.5 \text{ m}$$

$= 0.1333 + 2.5 = 2.633$ m from free surface.

Vertical force, F_y, exerted by water is given by equation (3.18)

F_y = Weight of water supported by AB upto free surface

= Weight of portion $DABOC$

= Weight of $DAOC$ + Weight of water AOB

$= \rho g$ [Volume of $DAOC$ + Volume of AOB]

$$= 1000 \times 9.81 \left[AD \times AO \times 1 + \frac{\pi}{4}(AO)^2 \times 1\right]$$

$$= 1000 \times 9.81 \left[1.5 \times 2.0 \times 1 + \frac{\pi}{4} \times 2^2 \times 1\right]$$

$= 1000 \times 9.81\ [3.0 + \pi]$N $=$ **60249.1 N. Ans.**

Problem 3.23 *Fig. 3.30 shows a gate having a quadrant shape of radius 2 m. Find the resultant force due to water per metre length of the gate. Find also the angle at which the total force will act.*

Solution. Given :

Radius of gate $= 2$ m

Width of gate $= 1$ m

Horizontal Force

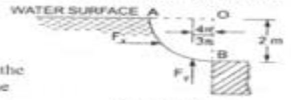


Fig. 3.30

F_x = Force on the projected area of the curved surface on vertical plane

$= \text{Force on } BO = \rho g A \bar{h}$

where $A = \text{Area of } BO = 2 \times 1 = 2 \text{ m}^2$, $\bar{h} = \frac{1}{2} \times 2 = 1$ m ;

$F_x = 1000 \times 9.81 \times 2 \times 1 = 19620$ N

This will act at a depth of $\frac{2}{3} \times 2 = \frac{4}{3}$ m from free surface of liquid.

Vertical Force, F_y

F_y = Weight of water (imagined) supported by AB

$= \rho g \times \text{Area of } AOB \times 1.0$

$= 1000 \times 9.81 \times \frac{\pi}{4} (2)^2 \times 1.0 = 30819$ N

This will act at a distance of $\frac{4R}{3\pi} = \frac{4 \times 2.0}{3\pi} = \mathbf{0.848}$ **m** from OB.

$\therefore$ Resultant force, F is given by

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{19620 + 30819} = \sqrt{384944400 + 949810761}$$

$= \mathbf{36534.4}$ **N. Ans.**

The angle made by the resultant with horizontal is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{30819}{19620} = 1.5708$$

$\therefore \quad \theta = \tan^{-1} 1.5708 = \mathbf{57° 31'}$. **Ans.**

Problem 3.26 *Calculate the horizontal and vertical components of the water pressure exerted on a tainter gate of radius 8 m as shown in Fig. 3.33. Take width of gate unity.*

Solution. The horizontal component of water pressure is given by

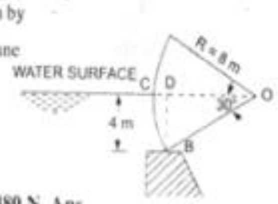


Fig. 3.33

$F_x = \rho g A \bar{h}$ = Force on the area projected on vertical plane

= Force on the vertical area of BD

where $A = BD \times \text{Width of gate} = 4.0 \times 1 = 4.0$ m

$\bar{h} = \frac{1}{2} \times 4 = 2$ m

$F_x = 1000 \times 9.81 \times 4.0 \times 2.0 = \mathbf{78480}$ **N. Ans.**

Vertical component of the water pressure is given by

F_y = Weight of water supported or enclosed (imaginary) by curved surface CB

= Weight of water in the portion $CBDC$

$= \rho g \times$ [Area of portion $CBDC$] $\times$ Width of gate

$= \rho g \times$ [Area of sector CBO − Area of the triangle BOD] $\times 1$

$$= 1000 \times 9.81 \times \left[\frac{30}{360} \times \pi R^2 - \frac{BD \times DO}{2}\right]$$

$$= 9810 \times \left[\frac{1}{12} \pi \times 8^2 - \frac{4.0 \times 8.8 \cos 30°}{2}\right]$$

$[\because DO = BO \cos 30° = 8 \times \cos 30°]$

$= 9810 \times [16.755 - 13.856] = \mathbf{28439}$ **N. Ans.**

Problem 3.31 *A cylinder 3 m in diameter and 4 m long retains water on one side. The cylinder is supported as shown in Fig. 3.39. Determine the horizontal reaction at A and the vertical reaction at B. The cylinder weighs 196.2 kN. Ignore friction.*

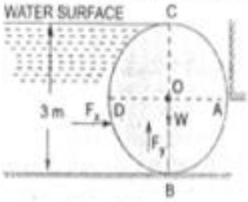


Fig. 3.39

Solution. Given :

Dia. of cylinder $= 3$ m

Length of cylinder $= 4$ m

Weight of cylinder, $W = 196.2$ kN $= 196200$ N

Horizontal force exerted by water

F_x = Force on vertical area BOC

$= \rho g A \bar{h}$

where $A = BOC \times l = 3 \times 4 = 12\ m^2$, $\bar{h} = \frac{1}{2} \times 3 = 1.5$ m

$F_x = 1000 \times 9.81 \times 12 \times 1.5 = 176580$ N

The vertical force exerted by water

F_y = Weight of water enclosed in $BDCOB$

$$= \rho g \times \left(\frac{\pi}{2} R^2\right) \times l = 1000 \times 9.81 \times \frac{\pi}{2} \times (1.5)^2 \times 4 = 138684 \text{ N}$$

Force F_y is acting in the upward direction.

For the equilibrium of cylinder

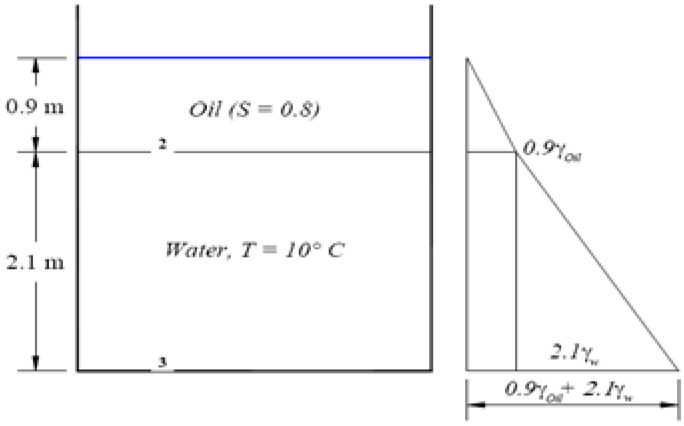
Horizontal reaction at $A = F_x = 176580$ N

Vertical reaction at B = Weight of cylinder $- F_y$

$= 196200 - 138684 =$ **57516 N. Ans.**

Example

Given: A tank full of water and oil (S_{oil} = 0.80), as shown.
Find: The pressure at the oil/water interface and the bottom of the tank.



Example 1:

A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied is 500 N at the plunger

Solution. Given :

Dia. of ram, $D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $F = 500 \text{ N}$

Find weight lifted $= W$

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \text{ m}^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = .00159 \text{ m}^2$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} \text{ N/m}^2.$$

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{.00159} = 314465.4 \text{ N/m}^2$$

But pressure intensity at ram $= \dfrac{\text{Weight}}{\text{Area of ram}} = \dfrac{W}{A} = \dfrac{W}{.07068} \text{ N/m}^2$

$$\frac{W}{.07068} = 314465.4$$

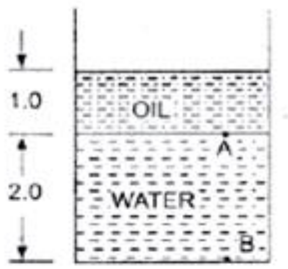
$\therefore$ Weight $= 314465.4 \times .07068 = 22222 \text{ N} =$ **22.222 kN. Ans.**

Problem :

An open tank contains water up to a depth of 2m and above it an oil of sp.gr. 0.9 for a depth of 1 m. Find the pressure intensity :

a) At the interface of the two liquids

b) At the bottom of the tank



Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m^3

Density of oil, ρ_2 = Sp. gr. of oil × Density of water $= 0.9 \times 1000 = 900$ kg/m^3

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

P= γ h=γ Z

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$

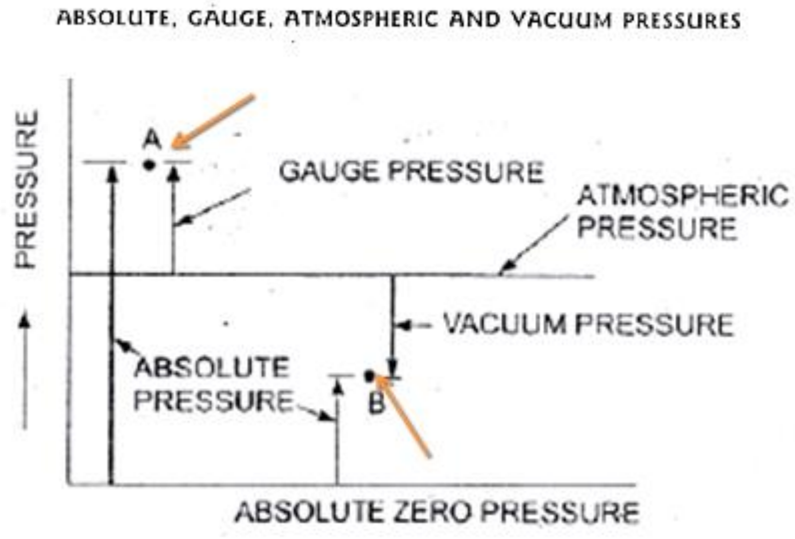
$$= 900 \times 9.81 \times 1.0$$

$$= 8829 \frac{N}{m^2} = \frac{8829}{10^4} = \mathbf{0.8829\ N/cm^2}.$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620 = 28449\ N/m^2 = \frac{28449}{10^4}\ N/cm^2 = \mathbf{2.8449\ N/cm^2}.$$



The atmospheric pressure at sea level at 15°C is 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit.

The pressure on a fluid is measured in two different systems.
a) It is measured above the absolute zero or complete vacuum and it is called the absolute pressure
b) Pressure is measured above the atmospheric pressure and it is called gauge pressure.
Thus:

1. Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. Gauge pressure is defined as the pressure which is measured with help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum . The atmospheric pressure on the scale is marked as zero.

3. Vacuum pressure is defined as the pressure below the atmospheric pressure.

(*i*) Absolute pressure
= Atmospheric pressure + Gauge pressure

$$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure
= Atmospheric pressure – Absolute pressure.

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Problem: What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53 x 10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury ? the specific gravity of mercury is 13.6 and density of water is 1000 kg/m^3

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

$\therefore$ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6 × 1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

$$\therefore \quad p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \quad (\because Z_0 = 0.75)$$

$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by.

$$p = \rho_1 \times g \times Z_1$$

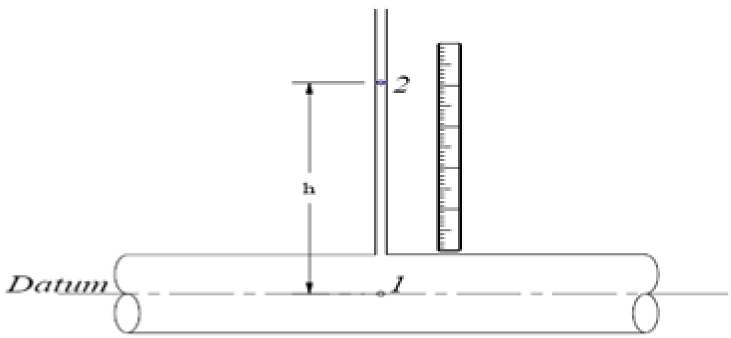
$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

$\therefore$ Gauge pressure, $\mathbf{p = 45028 \text{ N/m}^2}$. **Ans**

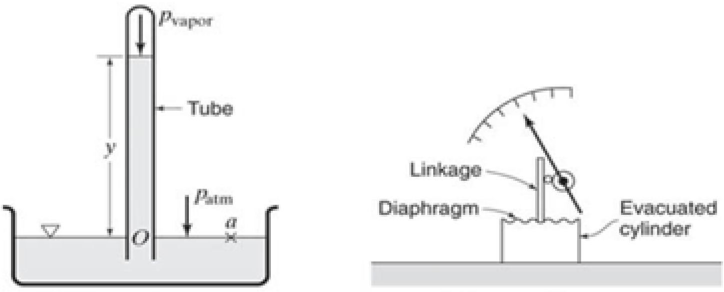
Now absolute pressure = Gauge pressure + Atmospheric pressure

$$= 45028 + 100062 = \mathbf{145090 \text{ N/m}^2}. \text{ Ans.}$$

Pressure Measurements



1- Barometer



(*a*) Mercury barometer (*b*) Aneroid barometer

Apparatus used to measure pressure; derived from the Greek "baros" meaning, "weight". Created by **Evangelista Torricelli** in 1646 who inverted a tube filled with mercury (Hg) into a dish until the force of the Hg inside the tube balanced the force of the atmosphere on the surface of the liquid outside the tube.

The two points O, and a are in the same elevation, then $p_o = p_a$

$p_o = \gamma_{HG} \times y + p_{vapor}$ (p_{vapor} can be neglected)

$p_o = \gamma_{HG} \times y$, and $p_a = p_{atm}$ ➔ $p_{atm} = \gamma_{HG} y$ (y = 760 mm of HG)

Atmospheric pressure units

	Unit	Abbreviation	Standard Atmospheric Magnitude (at Sea-level)	
Metric	Pascal	Pa	101325	~100000
	Bar	Bar	1.01325	~1
	Millibar	mb	1013.25	~1000
	Torr	Torr	760	
	Millimeters of Mercury	mmHg	760	

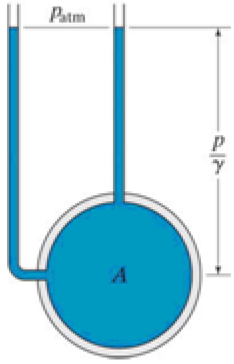
2- Piezometer Column

It can only measure positive pressure (greater than atmospheric pressure)

$p_A = \gamma h$

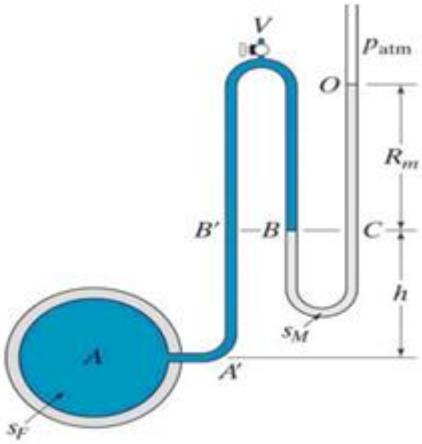
$h = \frac{p_A}{\gamma}$

The height in the piezometer can be very large depending on the liquid pressure; so, it is only used with moderate pressure values.



3- U-Tube manometer

It comes over the problems of the piezometer. It can measure negative pressure, and it can also be used to measure the pressure of gases.



a) Positive pressure

Assume:

s_M Specific gravity of the manometer fluid

$s_M = \frac{\gamma_M}{\gamma_w}$

γ_M unit weight of the manometer fluid (usually Mercury)

$$s_{HG} = \frac{\gamma_{HG}}{\gamma_w} = 13.56$$

s_f Specific gravity of the fluid under pressure

$s_f = \frac{\gamma_f}{\gamma_w}$

R_m Manometer reading (vertical distance from the top of manometer fluid surface in contact with air, and the interface between manometer fluid, and the fluid under pressure)

Datum at C:

$p_B = \gamma_m R_m$ $p_B = p_{B'}$

$p_{B'} = p_A - \gamma_f h$

$p_A - \gamma_f h = \gamma_m R_m$

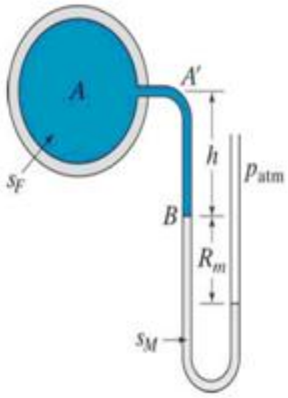
$$p_A = \gamma_f h + \gamma_m R_m$$

(Divide by γ_f)

$\frac{p_A}{\gamma_f} = h + \frac{\gamma_m}{\gamma_f} R_m$ $\frac{p_A}{\gamma_f} = h + \frac{\gamma_m}{\gamma_w}\frac{\gamma_w}{\gamma_f} R_m$

$$\frac{p_A}{\gamma_f} = h + \left(\frac{s_m}{s_f}\right) R_m$$

b) Negative pressure



Datum at B:

$p_B = \gamma_f h + p_A \quad \rightarrow \quad p_A = p_B - \gamma_f h$

$0 = \gamma_m R_m + p_B \quad \rightarrow \quad p_B = -\gamma_m R_m$

$p_A = -\gamma_m R_m - \gamma_f h$

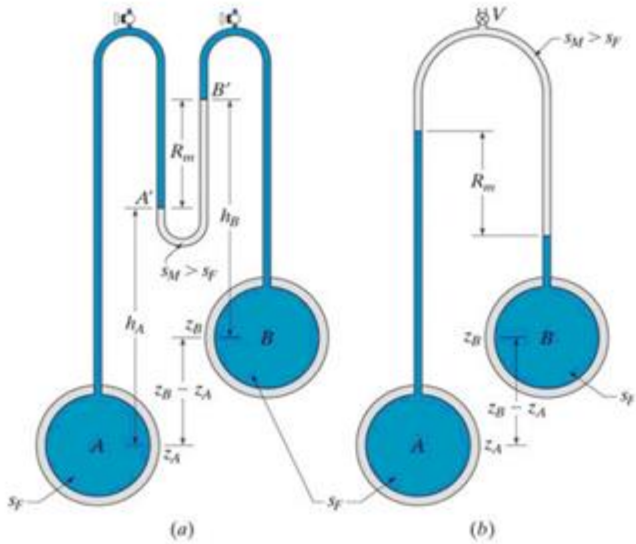
$$p_A = -(\gamma_f h + \gamma_m R_m)$$

(Divide by γ_f)

$$\frac{p_A}{\gamma_f} = -\left(h + \frac{\gamma_m}{\gamma_f} R_m\right) \qquad \frac{p_A}{\gamma_f} = -\left(h + \frac{\gamma_m}{\gamma_w}\frac{\gamma_w}{\gamma_f} R_m\right)$$

$$\frac{p_A}{\gamma_f} = -\left[h + \left(\frac{s_m}{s_f}\right) R_m\right]$$

Differential manometer



The only difference with the differential manometer is that it's connected to another pipeline instead of being subjected to atmospheric pressure.

a) $p_A = \gamma_f h_A + p_{x}$ $\quad\quad p_{x} = \gamma_m R_m + p_{y}$ $\quad\quad p_{y} = p_B - \gamma_f h_B$

$$p_A = \gamma_f h_A + \gamma_m R_m + p_B - \gamma_f h_B$$

$$p_A - p_B = \gamma_f h_A + \gamma_m R_m - \gamma_f h_B$$

$$\frac{p_A - p_B}{\gamma_f} = h_A + \frac{\gamma_m}{\gamma_f} R_m - h_B$$

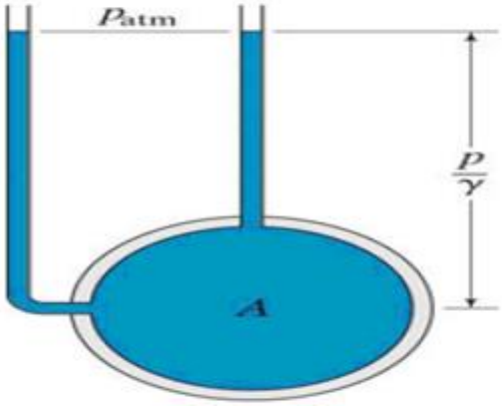
$$\frac{p_A - p_B}{\gamma_f} = (h_A - h_B) + \frac{s_m}{s_f} R_m$$

$h_A + R_m = (z_B - z_A) + h_B$ $\quad\quad \rightarrow h_A - h_B = (z_B - z_A) - R_m$

$$\frac{p_A - p_B}{\gamma_f} = (z_B - z_A) - R_m + \frac{s_m}{s_f} R_m$$

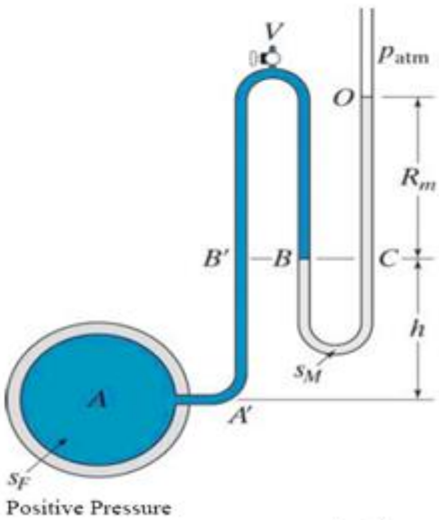
$$\frac{p_A - p_B}{\gamma_f} = (z_B - z_A) + \left(\frac{s_m}{s_f} - 1\right) R_m$$

Summary



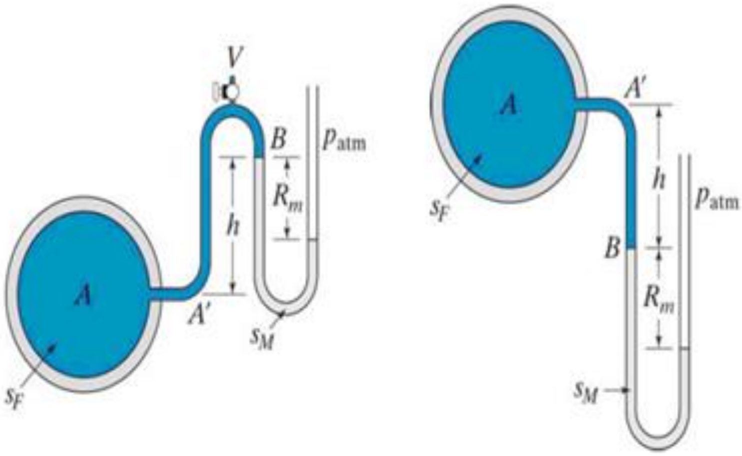
Piezometer

$p_A = \gamma h$

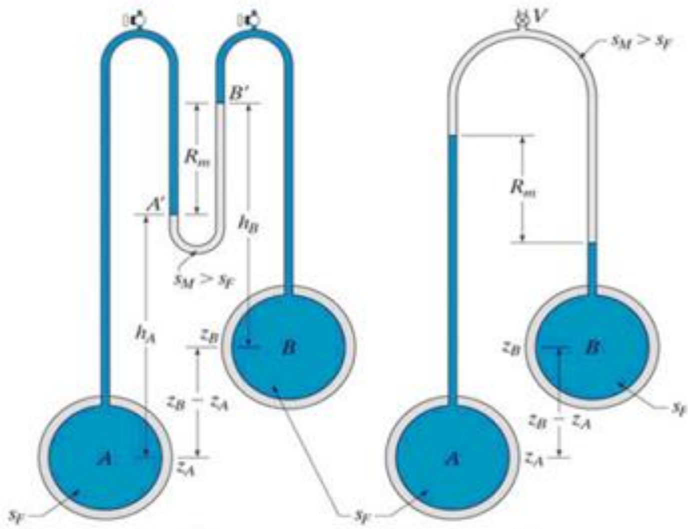


Positive Pressure

$p_A = \gamma_f h + \gamma_m R_m$ $\quad\quad$ $\frac{p_A}{\gamma_f} = h + \left(\frac{s_m}{s_f}\right) R_m$



$$p_A = -(\gamma_f h + \gamma_m R_m) \qquad \frac{p_A}{\gamma_f} = -\left[h + \left(\frac{s_m}{s_f}\right)R_m\right]$$



(a) (b)

$$\frac{p_A - p_B}{\gamma_f} = (h_A - h_B) + \frac{s_m}{s_f} R_m$$

$$\frac{p_A - p_B}{\gamma_f} = (z_B - z_A) + \left(\frac{s_m}{s_f} - 1\right) R_m$$

Example :1 The right limb of a simple U-tube manometer containing mercury is open To the atmosphere while the left limb is connected to a pipe in which a fluid of sp.gr 0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in right limb . Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900$ kg/m^3

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000$ kg/m^3

Difference of mercury level $h_2 = 20$ cm $= 0.2$ m

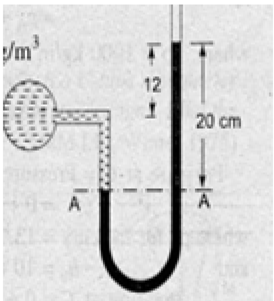
Height of fluid from A-A, $h_1 = 20 - 12 = 8$ cm $= 0.08$ m

Let p = Pressure of fluid in pipe

Equating the pressure above A-A, we get

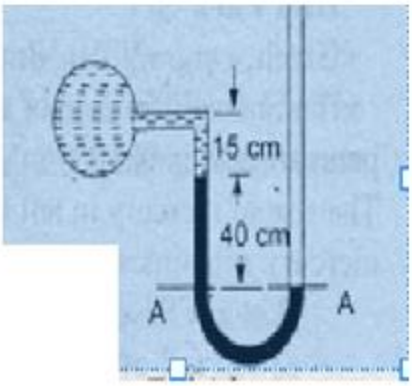
$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$



$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$
$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \quad \textbf{Ans.}$$

Problem 2.10 *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*



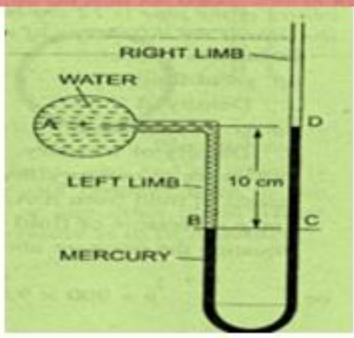
Final Answer

$$P = -\{\rho_2 g\, h_2 \;+\; \rho_1\, g\, h_1\}$$

P= -(13.6 x 1000 x 9.81 x0.4 + 800 x 9.81x 0.15)

=-(53366.4 + 1177.2)= - 54543.6 $\frac{N}{m^2}$

Problem 2.11 *A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m², calculate the new difference in the level of mercury. Sketch the arrangements in both cases.* (A.M.I.E., Winter 1989)



Solution. Given :

Difference of mercury $= 10 \text{ cm} = 0.1 \text{ m}$

The arrangement is shown in Fig. 2.11 (*a*)

Let p_A = (pressure of water in pipe line (*i.e.*, at point *A*)

The points *B* and *C* lie on the same horizontal line. Hence pressure at *B* should be equal to pressure at *C*. But pressure at *B*

= Pressure at *A* + Pressure due to 10 cm (or 0.1 m) of water

$$= p_A + \rho \times g \times h$$

where $\rho = 1000 \text{ kg/m}^3$ and $h = 0.1$ m

$$= p_A + 1000 \times 9.81 \times 0.1$$

$$= p_A + 981 \text{ N/m}^2 \quad \ldots(i)$$

Pressure at *C* = Pressure at *D* + Pressure due to 10 cm of mercury

$$= 0 + \rho_0 \times g \times h_0$$

where ρ_0 for mercury $= 13.6 \times 1000 \text{ kg/m}^3$

and $h_0 = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore$ Pressure at $C = 0 + (13.6 \times 1000) \times 9.81 \times 0.1$

$$= 13341.6 \text{ N} \quad \ldots(ii)$$

But pressure at *B* is equal to pressure at *C*. Hence equating the equations (*i*) and (*ii*), we get

$$p_A + 981 = 13341.6$$

$$\therefore \quad p_A = 13341.6 - 981$$

$$= 12360.6 \frac{\text{N}}{\text{m}^2}. \text{ Ans.}$$

IInd Part

Given, $p_A = 9810\ N/m^2$

Find new difference of mercury level. The arrangement is shown in Fig. 2.11 (*b*). In this case the pressure at *A* is 9810 N/m^2 which is less than the 12360.6 N/m^2. Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x= rise of mercury in left limb in cm
Then fall of mercury in right limb = x cm
The points B,C,D show the initial conditions whereas points B*,C*and D*
Show the final conditions

The pressure at B^* = Pressure at C^*

or Pressure at *A* + Pressure due to $(10 - x)$ cm of water
= Pressure at D^* + Pressure due to $(10 - 2x)$ cm of mercury

or $p_A + \rho_1 \times g \times h_1 = p_{D^*} + \rho_2 \times g \times h_2$

or $1910 + 1000 \times 9.81 \times \left(\frac{10 - x}{100}\right)$

$= 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10 - 2x}{100}\right)$

Dividing by 9.81, we get

or $1000 + 100 - 10x = 1360 - 272x$

or $272x - 10x = 1360 - 1100$

or $262x = 260$

$\therefore$ $x = \frac{260}{262} = 0.992$ cm

$\therefore$ New difference of mercury $= 10 - 2x$ cm $= 10 - 2 \times 0.992$
$= \mathbf{8.016}$ **cm. Ans.**

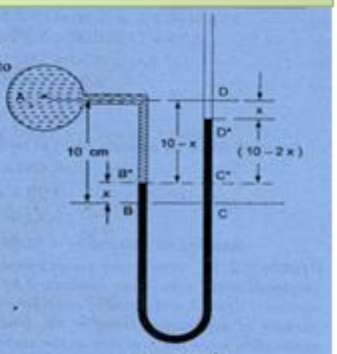
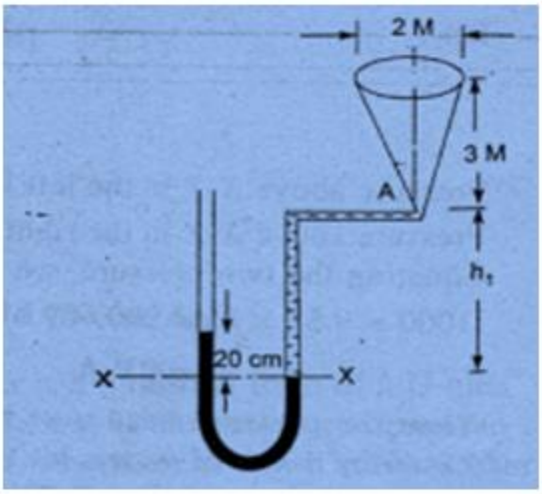


Fig. 2.11 (*b*)

Problem 2.12 *Fig. 2.12 shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.* (A.M.I.E., Winter 1975)



Solution. Vessel is empty. Given :

Difference of mercury level $h_2 = 20$ cm

Let h_1 = Height of water above X-X

Sp. gr. of mercury, $S_2 = 13.6$

Sp. gr. of water, $S_1 = 1.0$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Density of water, $\rho_1 = 1000$

Equating the pressure above datum line X-X, we have

$$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$$

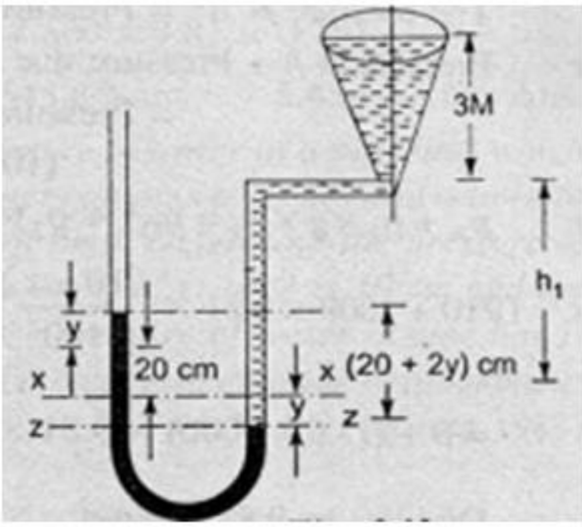
or $$13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$h_1 = 2.72 \text{ m of water.}$$

Vessel is full of water. When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be, *y* cm as shown in Fig. 2.13. The mercury will rise in the left by a distance of *y* cm. Now the datum line is *Z-Z*. Equating the pressure above the datum line *Z-Z*.

Pressure in left limb = Pressure in right limb

$$13.6 \times 1000 \times 9.81 \times (0.2 + 2y/100) = 1000 \times 9.81 \times (3 + h_1 + y/100)$$



or $\quad 13.6 \times (0.2 + 2y/100) = (3 + 2.72 + y/100) \quad (\because \; h_1 = 2.72 \text{ cm})$

or $\quad 2.72 + 27.2y/100 = 3 + 2.72 + y/100$

or $\quad (27.2y - y)/100 = 3.0$

or $\quad 26.2y = 3 \times 100 = 300$

$\therefore \quad y = \dfrac{300}{26.2} = 11.45 \text{ cm}$

The difference of mercury level in two limbs

$= (20 + 2y)$ cm of mercury

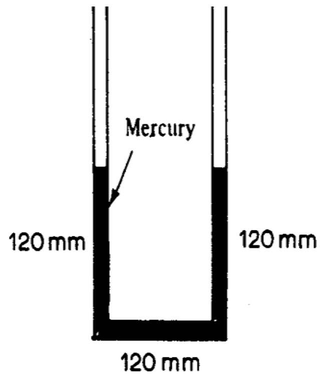
$= 20 + 2 \times 11.45 = 20 + 22.90$

$= 42.90$ cm of mercury

$\therefore$ Reading of manometer = **42.90 cm. Ans.**

problems

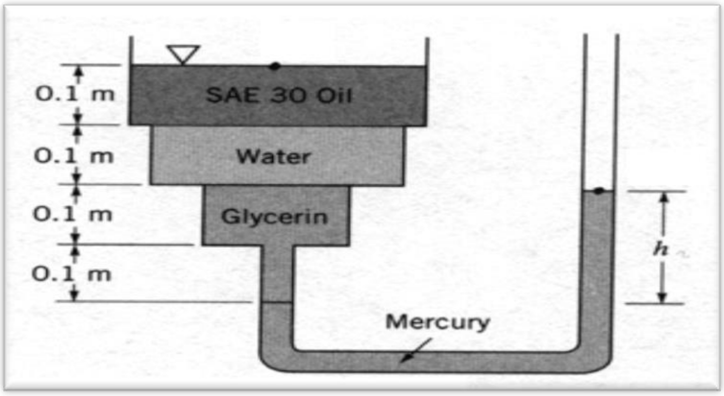
Q1) The U-tube shown below is 10 mm in diameter and contain mercury. If 12 ml of water is poured into the right-hand leg. What are the ultimate heights in the two legs.



Q2)

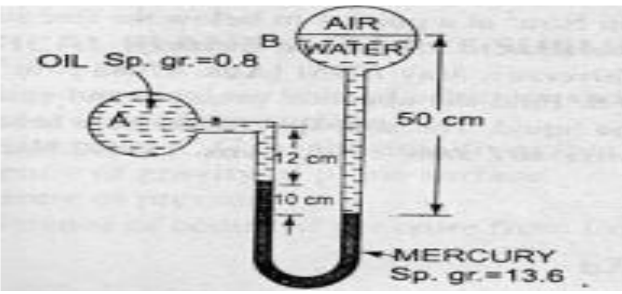
A Tank is constructed of a series of cylinders having diameters of 0.3, 0.25, and 0.15 m as shown in fig. below. The tank contains oil (s.g =0.89

), water and glycerin (s,g= 1.2) and mercury manometer is attached to the bottom as illustrated. Calculate the manometer reading (h).



Q3) A differential manometer is connected at the two points A and B as shown below. At B air pressure is 7.848 N/cm2 (abs). Find the absolute pressure at A.

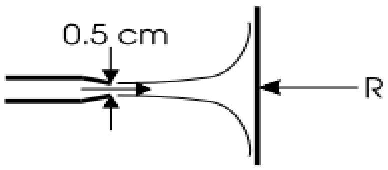
Ans. 6.91 N/cm2



Momentum Principle

Problem 6.1

Water at 20°C is discharged from a nozzle onto a plate as shown. The flow rate of the water is 0.001 m^3/s, and the diameter of the nozzle outlet is 0.5 cm. Find the force necessary to hold the plate in place.

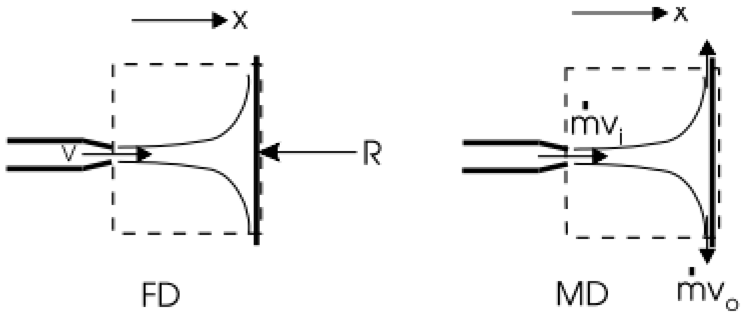


Solution

This is a one-dimensional, steady flow. Since the system is not accelerating, the velocities with respect to the nozzle and plate are inertial velocities. The momentum equation in the x-direction (horizontal direction) is

$$\sum F_x = \sum \dot{m}_o v_{o_x} - \sum \dot{m}_i v_{i_x}$$

Draw a control volume with the associated force and momentum diagrams.



From the force diagram

$$\sum F_x = -R$$

From the continuity equation, the mass flow in is equal to the mass flow out so

$$\dot{m}_o = \dot{m}_i = \dot{m}$$

The velocity at the inlet is V. The component of velocity in the x-direction at the outlet is zero, so the momentum flux is

$$\sum \dot{m}_o v_{o_x} - \sum \dot{m}_i v_{i_x} = -\dot{m}V$$

Equating the forces and momentum flux

$$-R = -\dot{m}V$$

or

$$R = \dot{m}V$$

The volume flow rate is 0.01 m^3/s, so the mass flow rate is $\dot{m} = \rho Q = 1000 \times 0.001 = 1$ kg/s. The velocity is

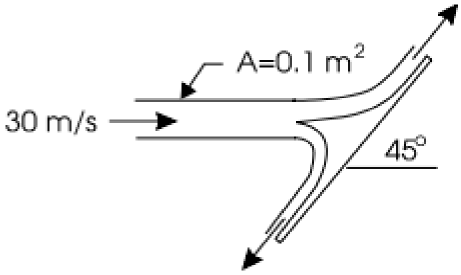
$$V = \frac{Q}{A} = \frac{0.001}{\frac{\pi}{4}(0.005)^2} = 50.9 \text{ m/s}$$

The restraining force is

$$R = 50.9 \times 1 = \underline{\underline{50.9 \text{ N}}}$$

Problem 2

A water jet with a velocity of 30 m/s impacts on a splitter plate so that $\frac{1}{4}$ of the water is deflected toward the bottom and $\frac{3}{4}$toward the top. The angle of the plate is 45^o. Find the force required to hold the plate stationary. Neglect the weight of the plate and water, and neglect viscous effects.



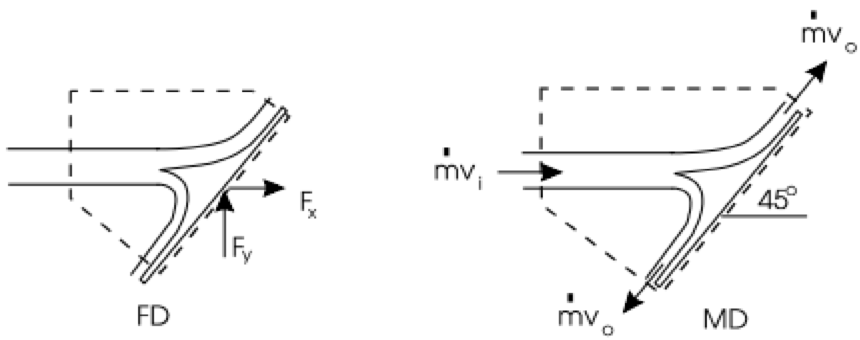
Solution

The pressure is constant on the free surface of the water. Because frictional effects are neglected, the Bernoulli equation is applicable. Without gravitational effects, the Bernoulli equation becomes

$$p + \frac{1}{2}\rho V^2 = \text{constant}$$

Since pressure is constant, the velocity will be constant. Therefore, each exit velocity is equal to the inlet velocity.

Momentum and force diagrams for this problem are



The forces acting on the control surface are

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

The momentum flux from the momentum diagram is

$$\sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i = \frac{3}{4}\dot{m}_i(30\mathbf{i}\cos 45 + 30\mathbf{j}\sin 45) + \frac{1}{4}\dot{m}_i(-30\mathbf{i}\cos 45 - 30\mathbf{j}\sin 45) - \dot{m}_i 30\mathbf{i}$$

Equating the force and momentum flux

$$F_x\mathbf{i} + F_y\mathbf{j} = \dot{m}_i(-19.4\mathbf{i}+10.6\mathbf{j})$$

The inlet mass flow rate is

$$\dot{m}_i = \rho AV = 1000 \times 0.1 \times 30 = 3000 \text{ kg/s}$$

The force vector evaluates to

$$F_x\mathbf{i} + F_y\mathbf{j} = -5.82 \times 10^4\mathbf{i} + 3.18 \times 10^4\mathbf{j} \text{ (N)}$$

Thus

$$F_x = \underline{\underline{-5.82 \times 10^4 \text{ N}}}$$
$$F_y = \underline{\underline{3.18 \times 10^4 \text{ N}}}$$

▶ 6.8 THE MOMENTUM EQUATION

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on a fluid mass '*m*' is given by the Newton's second law of motion,

$$F = m \times a$$

where *a* is the acceleration acting in the same direction as force *F*.

But $$a = \frac{dv}{dt}$$

∴ $$F = m\frac{dv}{dt}$$

$$= \frac{d(mv)}{dt} \quad \{m \text{ is constant and can be taken inside the differential}\}$$

∴ $$F = \frac{d(mv)}{dt} \qquad \text{...(6.15)}$$

Equation (6.15) is known as the momentum principle.

Equation (6.15) can be written as $F.dt = d(mv)$ ...(6.16)

which is known as the *impulse-momentum equation* and states that the impulse of a force *F* acting on a fluid of mass *m* in a short interval of time *dt* is equal to the change of momentum *d(mv)* in the direction of force.

scanned by Fahid

Force exerted by a flowing fluid on a Pipe-Bend

The impulse-momentum equation (6.16) is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections (1) and (2), as shown in Fig. 6.18.

Let v_1 = velocity of flow at section (1),
p_1 = pressure intensity at section (1),
A_1 = area of cross-section of pipe at section (1) and
v_2, p_2, A_2 = corresponding values of velocity, pressure and area at section (2).

Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in x-and y-directions respectively. Then the force exerted by the bend on the fluid in the directions of x and y will be equal to F_x and F_y but in the opposite directions. Hence component of the force exerted by bend on the fluid in the x-direction $= -F_x$ and in the direction of $y = -F_y$. The other external forces acting on the fluid are p_1A_1 and p_2A_2 on the sections (1) and (2) respectively. Then momentum equation in x-direction is given by

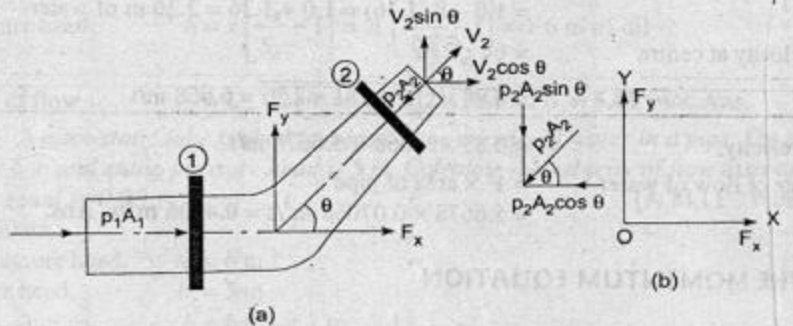


Fig. 6.18 *Forces on bend.*

Net force acting on fluid in the direction of x = Rate of change of momentum in x-direction

$$\therefore \quad p_1A_1 - p_2A_2\cos\theta - F_x = \text{(Mass per sec) (change of velocity)}$$

$$= \rho Q \text{ (Final velocity in the direction of } x - \text{Initial velocity in the direction of } x)$$

$$= \rho Q (V_2 \cos\theta - V_1) \qquad ...(6.17)$$

$$\therefore \quad F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1A_1 - p_2A_2\cos\theta \qquad ...(6.18)$$

Similarly the momentum equation in y-direction gives

$$0 - p_2A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0) \qquad ...(6.19)$$

$$\therefore \quad F_y = \rho Q (-V_2 \sin\theta) - p_2A_2 \sin\theta \qquad ...(6.20)$$

Now the resultant force (F_R) acting on the bend

$$= \sqrt{F_x^2 + F_y^2} \qquad ...(6.21)$$

And the angle made by the resultant force with horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x} \qquad ...(6.22)$$

Problem 6.29 *A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm^2 and rate of flow of water is 600 litres/s.*

Solution. Given :

Angle of bend, $\theta = 45°$

Dia. at inlet, $D_1 = 600 \text{ mm} = 0.6 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.6)^2$

$= 0.2827 \text{ m}^2$

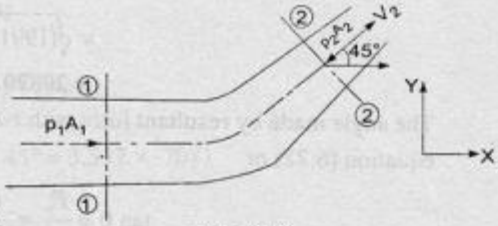


Fig. 6.19

Dia. at outlet, $D_2 = 300 \text{ mm} = 0.30 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} (3)^2 = 0.07068 \text{ m}^2$

Pressure at inlet, $p_1 = 8.829 \text{ N/cm}^2 = 8.829 \times 10^4 \text{ N/m}^2$

$Q = 600 \text{ lit/s} = 0.6 \text{ m}^3\text{/s}$

$$V_1 = \frac{Q}{A_1} = \frac{0.6}{.2827} = 2.122 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.07068} = 8.488 \text{ m/s.}$$

Applying Bernoulli's equation at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$

$\therefore$ $$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \quad \text{or} \quad \frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{8.488^2}{2 \times 9.81}$$

$$9 + .2295 = p_2/\rho g + 3.672$$

$\therefore$ $$\frac{p_2}{\rho g} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$$

$\therefore$ $$p_2 = 5.5575 \times 1000 \times 9.81 \text{ N/m}^2 = 5.45 \times 10^4 \text{ N/m}^2$$

Forces on the bend in x- and y-directions are given by equations (6.18) and (6.20) as

$$F_x = \rho Q\,[v_1 - v_2 \cos \theta] + p_1A_1 - p_2A_2 \cos \theta$$
$$= 1000 \times 0.6\,[2.122 - 8.488 \cos 45°]$$
$$+ 8.829 \times 10^4 \times .2827 - 5.45 \times 10^4 \times .07068 \times \cos 45°$$
$$= -2327.9 + 24959.6 - 2720.3 = 24959.6 - 5048.2$$
$$= 19911.4 \text{ N}$$

and $$F_y = \rho Q\,[- V_2 \sin \theta] - p_2A_2 \sin \theta$$
$$= 1000 \times 0.6\,[- 8.488 \sin 45°] - 5.45 \times 10^4 \times .07068 \times \sin 45°$$
$$= -3601.1 - 2721.1 = -6322.2 \text{ N}$$

–ve sign means F_y is acting in the downward direction

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(19911.4)^2 + (-6322.2)^2}$$
$$= \mathbf{20890.9\ N.\ Ans.}$$

The angle made by resultant force with x-axis is given by equation (6.22) or

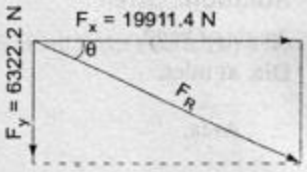


Fig. 6.20

$$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$$

$\therefore$ $$\theta = \tan^{-1} .3175 = \mathbf{17° 36'. Ans.}$$

Problem 6.30 *250 litres/s of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135° (that is change from initial to final direction is 135°), find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm².*

(A.M.I.E., Winter, 1974)

Solution. Given :

Pressure, $p_1 = p_2 = 39.24\ \text{N/cm}^2 = 39.24 \times 10^4\ \text{N/m}^2$

Discharge, $Q = 250\ \text{litres/s} = 0.25\ \text{m}^3\text{/s}$

Dia. of bend at inlet and outlet, $D_1 = D_2 = 300\ \text{mm} = 0.3\ \text{m}$

$\therefore$ Area, $A_1 = A_2 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068\ \text{m}^2$

Velocity of water at (1) and (2), $V = V_1 = V_2 = \frac{Q}{\text{Area}} = \frac{0.25}{.07068} = 3.537\ \text{m/s}.$

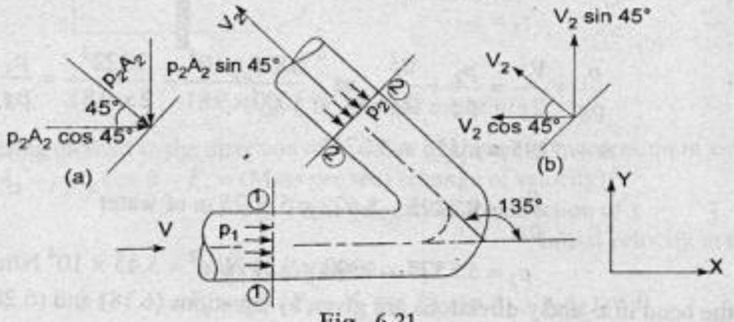


Fig. 6.21

Force along x-axis

$$= F_x = \rho Q[V_{1x} - V_{2x}] + p_{1x}A_1 + p_{2x}A_2$$

where V_{1x} = initial velocity in the direction of x = 3.537 m/s

V_{2x} = final velocity in the direction of $x = -V_2 \cos 45° = -3.537 \times .7071$

p_{1x} = pressure at (1) in x-direction
$= 39.24\ \text{N/cm}^2 = 39.24 \times 10^4\ \text{N/m}^2$

p_{2x} = pressure at (2) in x-direction
$= p_2 \cos 45° = 39.24 \times 10^4 \times .7071$

$$\therefore \quad F_x = 1000 \times .25[3.537 - (-3.537 \times .7071)] + 39.24 \times 10^4 \times .07068 + 39.24 \times 10^4 \times .07068 \times .7071$$
$$= 1000 \times .25[3.537 + 3.537 \times .7071] + 39.24 \times 10^4 \times .07068\ [1 + .7071]$$

$$= 1509.4 + 47346 = 48855.4 \text{ N}$$

Force along y-axis

$$= F_y = \rho Q[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$$

where V_{1y} = initial velocity in y-direction = 0

V_{2y} = final velocity in y-direction $= -V_2 \sin 45° = 3.537 \times .7071$

$(p_1A_1)_y$ = pressure force in y-direction = 0

$(p_2A_2)_y$ = pressure force at (2) in y-direction

$$= -p_2A_2 \sin 45° = -39.24 \times 10^4 \times .07068 \times .7071$$

$$\therefore \quad F_y = 1000 \times .25[0 - 3.537 \times .7071] + 0 + (-39.24 \times 10^4 \times .07068 \times .7071)$$

$$= -625.2 - 19611.1 = -20236.3 \text{ N}$$

– ve sign means F_y is acting in the downward direction

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{48855.4^2 + 20236.3^2}$$

$$= \mathbf{52880.6 \text{ N}}$$

The direction of the resultant force F_R, with the x-axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{20236.3}{48855.4} = 0.4142$$

$$\therefore \quad \theta = \mathbf{22° \, 30'.\ Ans.}$$

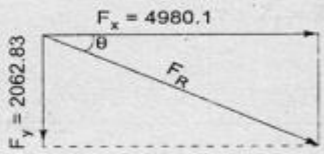


Fig. 6.22

Problem 6.31 *A 300 mm diameter pipe carries water under a head of 20 metres with a velocity of 3.5 m/s. If the axis of the pipe turns through 45°, find the magnitude and direction of the resultant force at the bend.* (A.M.I.E., Summer, 1978)

Solution. Given :

Dia. of bend, $D = D_1 = D_2 = 300 \text{ mm} = 0.30 \text{ m}$

$\therefore$ Area, $A = A_1 = A_2 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$

Velocity, $V = V_1 = V_2 = 3.5 \text{ m/s}$

$\theta = 45°$

Discharge, $Q = A \times V = 0.07068 \times 3.5 = 0.2475 \text{ m}^3/\text{s}$

Pressure head $= 20$ m of water or $\frac{p}{\rho g} = 20$ m of water

$$\therefore \quad p = 20 \times \rho g = 20 \times 1000 \times 9.81 \text{ N/m}^2 = 196200 \text{ N/m}^2$$

$\therefore$ Pressure intensity, $p = p_1 = p_2 = 196200 \text{ N/m}^2$

Now $V_{1x} = 3.5 \text{ m/s}.\ V_{2x} = V_2 \cos 45° = 3.5 \times .7071$

$V_{1y} = 0,\ V_{2y} = V_2 \sin 45° = 3.5 \times 7071$

$(p_1A_1)_x = p_1A_1 = 196200 \times .07068,\ (p_1A_1)_y = 0$

$(p_2A_2)_x = -p_2A_2 \cos 45°,\ (p_2A_2)_y = -p_2A_2 \sin 45°$

Force along x-axis, $F_x = \rho Q[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

$$= 1000 \times .2475[3.5 - 3.5 \times .7071] + 196200 \times .07068 - p_2A_2 \times \cos 45°$$

Problem 6.31 *A 300 mm diameter pipe carries water under a head of 20 metres with a velocity of 3.5 m/s. If the axis of the pipe turns through 45°, find the magnitude and direction of the resultant force at the bend.* (A.M.I.E., Summer, 1978)

Solution. Given :

Dia. of bend, $D = D_1 = D_2 = 300 \text{ mm} = 0.30 \text{ m}$

$\therefore$ Area, $A = A_1 = A_2 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$

Velocity, $V = V_1 = V_2 = 3.5 \text{ m/s}$

$\theta = 45°$

Discharge, $Q = A \times V = 0.07068 \times 3.5 = 0.2475 \text{ m}^3/\text{s}$

Pressure head $= 20$ m of water or $\frac{p}{\rho g} = 20$ m of water

$\therefore$ $p = 20 \times \rho g = 20 \times 1000 \times 9.81 \text{ N/m}^2 = 196200 \text{ N/m}^2$

$\therefore$ Pressure intensity, $p = p_1 = p_2 = 196200 \text{ N/m}^2$

Now $V_{1x} = 3.5 \text{ m/s}, \; V_{2x} = V_2 \cos 45° = 3.5 \times .7071$

$V_{1y} = 0, \; V_{2y} = V_2 \sin 45° = 3.5 \times .7071$

$(p_1A_1)_x = p_1A_1 = 196200 \times .07068, \; (p_1A_1)_y = 0$

$(p_2A_2)_x = -p_2A_2 \cos 45°, \; (p_2A_2)_y = -p_2A_2 \sin 45°$

Force along x-axis, $F_x = \rho Q[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

$= 1000 \times .2475[3.5 - 3.5 \times .7071] + 196200 \times .07068 - p_2A_2 \times \cos 45°$

$= 253.68 + 196200 \times .07068 - 196200 \times .07068 \times 0.7071$

$= 253.68 + 13871.34 - 9808.04 = 4316.98$ N

Force along y-axis, $F_y = \rho Q\,[v_1y - v_2y] + (p_1A_1)_y + (p_2A_2)_y$

$= 1000 \times .2475[0 - 3.5 \times .7071] + 0 + [-p_2A_2 \sin 45°]$

$= -612.44 - 196200 \times .07068 \times .7071$

$= -612.44 - 9808 = -10420.44$ N

$\therefore$ Resultant force $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(4316.98)^2 + (10420.44)^2} =$ **11279 N. Ans.**

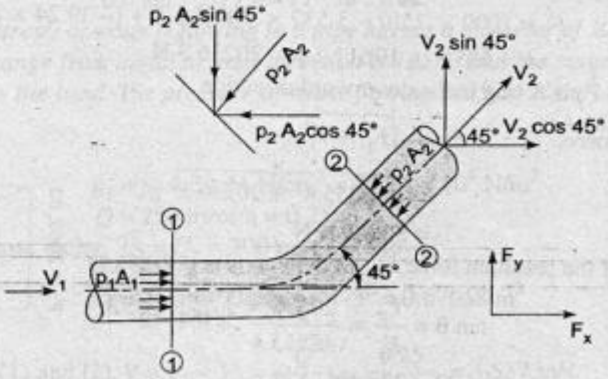


Fig. 6.23

The angle made by F_R with x-axis

$$\tan\theta = \frac{F_y}{F_x} = \frac{10420.44}{4316.98} = 2.411$$

$\therefore$ $\theta = \tan^{-1} 2.411 =$ **67° 28′. Ans.**

Problem 6.32 *In a 45° bend a rectangular air duct of 1 m^2 cross-sectional area is gradually reduced to 0.5 m^2 area. Find the magnitude and direction of the force required to hold the duct in position if the velocity of flow at the 1 m^2 section is 10 m/s, and pressure is 2.943 N/cm^2. Take density of air as 1.16 kg/m^3.* (A.M.I.E., Winter, 1980)

Solution. Given :

Area at section (1), $A_1 = 1 \text{ m}^2$

Area at section (2), $A_2 = 0.5 \text{ m}^2$

Velocity at section (1), $V_1 = 10 \text{ m/s}$

Pressure at section (1), $p_1 = 2.943 \text{ N/cm}^2 = 2.943 \times 10^4 \text{ N/m}^2 = 29430 \text{ N/m}^2$

Density of air, $\rho = 1.16 \text{ kg/m}^3$

Applying continuity equation at sections (1) and (2)

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{1}{0.5} \times 10 = 20 \text{ m/s}$$

Discharge $Q = A_1V_1 = 1 \times 10 = 10 \text{ m}^3/\text{s}$

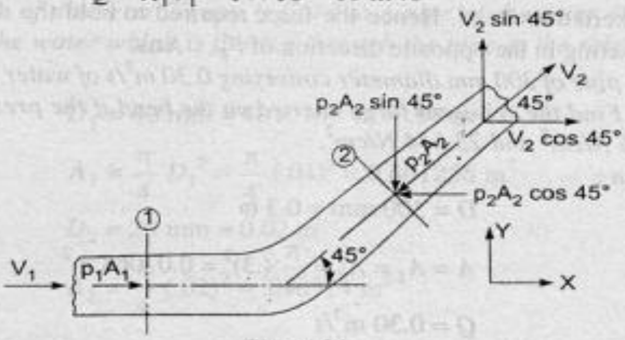


Fig. 6.24

Applying Bernoulli's equation at (1) and (2)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \qquad \{\because \quad Z_1 = Z_2\}$$

or $$\frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{10^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{20^2}{2 \times 9.81}$$

$$\therefore \quad \frac{p_2}{\rho g} = \frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{10^2}{2 \times 9.81} - \frac{20^2}{2 \times 9.81}$$

$$= 2586.2 + 5.0968 - 20.387 = 2570.90 \text{ m}$$

$$\therefore \quad p_2 = 2570.90 \times 1.16 \times 9.81 = 29255.8 \text{ N}$$

Force along x-axis, $F_x = \rho Q\,[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

where $A_{1x} = 10$ m/s, $V_{2x} = V_2 \cos 45° = 20 \times .7071$,

$(p_1A_1)_x = p_1A_1 = 29430 \times 1 = 29430$ N

and $(p_2A_2)_x = -p_2A_2 \cos 45° = -p_2A_2 \cos 45° = -29255.8 \times 0.5 \times .7071$

$$\therefore \quad F_x = 1.16 \times 10[10 - 20 \times .7071] + 29430 \times 1 - 29255.8 \times .5 \times .7071$$

$$= -48.04 + 29430 - 10343.37 = 0 - 19038.59 \text{ N}$$

Similarly force along y-axis, $F_y = \rho Q[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

where $V_{1y} = 0$, $V_{2y} = V_2 \sin 45° = 20 \times .7071 = 14.142$

$(p_1A_1)_y$
$= 0$, $(p_2A_2)_y = -p_2A_2 \sin 45° = -29255.8 \times .5 \times .7071 = -10343.37$

$$F_y = 1.16 \times 10\,[0 - 14.142] + 0 - 10343.37$$

$$= -164.05 - 10343.37 = -10507.42 \text{ N}$$

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19038.6)^2 + (10507.42)^2} = $ **21746.6 N. Ans.**

The direction of F_R with x-axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{10507.42}{19038.6} = 0.5519$$

$\therefore$ $\theta = \tan^{-1} .5519 = $ **28° 53′. Ans.**

F_R is the force exerted on bend. Hence the force required to hold the duct in position is equal to 21746.6 N but it is acting in the opposite direction of F_R. Ans.

Problem 6.33 *A pipe of 300 mm diameter conveying 0.30 m³/s of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and outlet of the bend are 24.525 N/cm² and 23.544 N/cm².*

Solution. Given :

Dia. of bend, $D = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A = A_1 = A_2 = \frac{\pi}{4}(.3)^2 = 0.07068 \text{ m}^2$

$\therefore$ Discharge, $Q = 0.30 \text{ m}^3/\text{s}$

$\therefore$ Velocity, $V = V_1 = V_2 = \frac{Q}{A} = \frac{0.30}{.07068} = 4.244 \text{ m/s}$

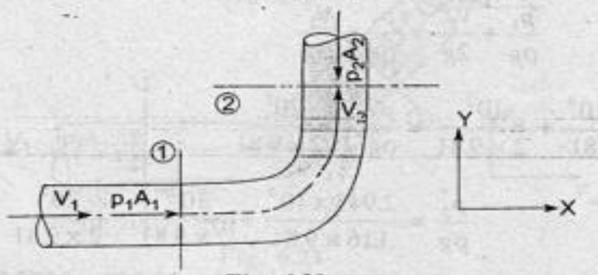


Fig. 6.25

Angle of bend, $\theta = 90°$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2 = 245250 \text{ N/m}^2$

$p_2 = 23.544 \text{ N/cm}^2 = 23.544 \times 10^4 \text{ N/m}^2 = 235440 \text{ N/m}^2$

Force of bend along x-axis $F_x = \rho Q\,[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

where $\rho = 1000$, $V_{1x} = V_1 = 4.244$ m/s, $V_{2x} = 0$

$(p_1A_1)_x = p_1A_1 = 245250 \times .07068$

$(p_2A_2)_x = 0$

$\therefore$ $F_x = 1000 \times 0.30\,[4.244 - 0] + 245250 \times .07068 + 0$

$= 1273.2 + 17334.3 = 18607.5 \text{ N}$

Force on bend along y-axis, $F_y = \rho Q\,[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

where $V_{1y} = 0$, $V_{2y} = V_2 = 4.244$ m/s

$(p_1A_1)_y = 0$, $(p_2A_2)_y = -p_2A_2 = -235440 \times .07068 = -16640.9$

$\therefore$ $F_y = 1000 \times 0.30[0 - 4.244] + 0 - 16640.9$

$= -1273.2 - 16640.9 = -17914.1 \text{ N}$

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(18607.5)^2 + (17914.1)^2} = -25829.3 \text{ N}$

and $\tan\theta = \frac{F_y}{F_x} = \frac{17914.1}{18607.5} = 0.9627$

$\therefore$ $\theta = 43° 54'$. **Ans.**

Problem 6.34 *A nozzle of diameter 20 mm is fitted to a pipe of diameter 40 mm. Find the force exerted by the nozzle on the water which is flowing through the pipe at the rate of 1.2 m^3/minute.*

Solution. Given :

Dia. of pipe, $D_1 = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} = .04 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(.04)^2 = 0.001256 \text{ m}^2$

Dia. of nozzle, $D_2 = 20 \text{ mm} = 0.02 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4}(.02)^2 = .000314 \text{ m}^2$

Discharge, $Q = 1.2 \text{ m}^3/\text{minute} = \frac{1.2}{60} \text{ m}^3/\text{s} = 0.02 \text{ m}^3/\text{s}$

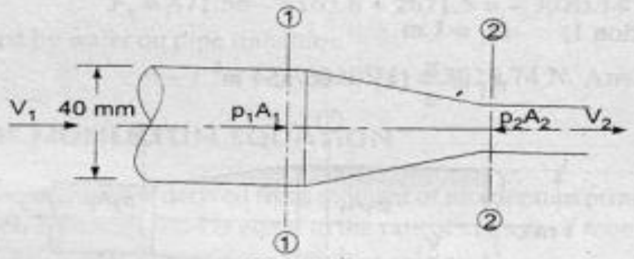


Fig. 6.26

Applying continuity equation at (1) and (2),

$$A_1V_1 = A_2V_2 = Q$$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.2}{.001256} = 15.92 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{0.2}{.000314} = 63.69 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Now $z_1 = z_2$, $\frac{p_2}{\rho g}$ = atmospheric pressure = 0

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{p_1}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{(63.69^2)}{2 \times 9.81} - \frac{(15.92^2)}{2 \times 9.81} = 206.749 - 12.917$$

$$= 193.83 \text{ m of water}$$

$$\therefore \quad p_1 = 193.83 \times 1000 \times 9.81 \frac{\text{N}}{\text{m}^2} = 1901472 \frac{\text{N}}{\text{m}^2}$$

Let the force exerted by the nozzle on water = F_x

Net force in the direction of x = rate of change of momentum in the direction of x

$$\therefore \quad p_1A_1 - p_2A_2 + F_x = \rho Q(V_2 - V_1)$$

where p_2 = atmospheric pressure = 0 and ρ = 1000

$\therefore$ $1901472 \times .001256 - 0 + F_x = 1000 \times 0.02(63.69 - 15.92)$ or $2388.24 + F_x = 916.15$

$$\therefore \quad F_x = -2388.24 + 916.15 = \mathbf{-1472.09.}$$ **Ans.**

Problem 6.35 *The diameter of a pipe gradually reduces from 1 m to 0.7 m as shown in Fig. 6.27. The pressure intensity at the centre-line of 1 m section 7.848 kN/m² and rate of flow of water through the pipe is 600 litres/s. Find the intensity of pressure at the centre-line of 0.7 m section. Also determine the force exerted by flowing water on transition of the pipe.*

Solution. Given :

Dia. of pipe at section 1, $D_1 = 1$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ m}^2$

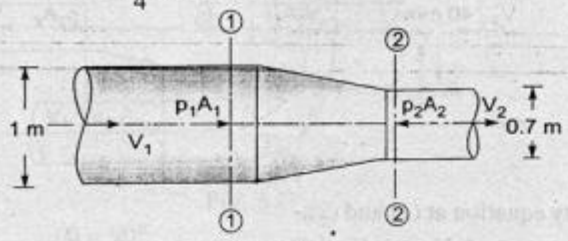


Fig. 6.27

Dia. of pipe at section 2, $D_2 = 0.7$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4}(0.7)^2 = 0.3848 \text{ m}^2$

Pressure at section 1, $p_1 = 7.848 \text{ kN/m}^2 = 7848 \text{ N/m}^2$

Discharge, $Q = 600 \text{ litres/s} = \frac{600}{1000} = 0.6 \text{ m}^3\text{/s}$

Applying continuity equation,

$$A_1V_1 = A_2V_2 = Q$$

$$\therefore \quad V_1 = \frac{Q}{A_2} = \frac{0.6}{0.7854} = 0.764 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.3854} = 1.55 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2),

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \qquad \{\because \text{ pipe is horizontal, } \therefore z_1 = z_2\}$$

or $$\frac{7848}{1000 \times 9.81} + \frac{(.764)^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{(1.55)^2}{2 \times 9.81}$$

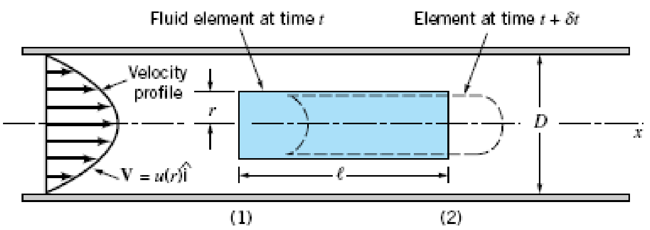
Flow in pipes

Laminar flow for real fluid

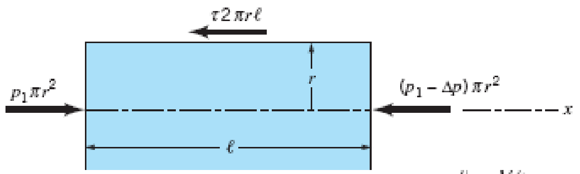
$$(p_1)\pi r^2 - (p_1 - \Delta p)\pi r^2 - (\tau)2\pi r\ell = 0$$

which can be simplified to give

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r} \tag{8.3}$$



■ FIGURE 8.7 Motion of a cylindrical fluid element within a pipe.



■ FIGURE 8.8 Free-body

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$

where the dimensionless quantity

$$f = \Delta p(D/\ell)/(\rho V^2/2)$$

$$\tau = \frac{2\tau_w r}{D}$$

$$\Delta p = \frac{4\ell\tau_w}{D}$$

is termed the *friction factor*, or sometimes the *Darcy friction factor* [H. P. G. Darcy (1803–1858)]. (This parameter should not be confused with the less-used Fanning friction factor, which is defined to be $f/4$. In this text we will use only the Darcy friction factor.) Thus, the friction factor for laminar fully developed pipe flow is simply

$$f = \frac{64}{\text{Re}} \tag{8.19}$$

By substituting the pressure drop in terms of the wall shear stress (Eq. 8.5), we obtain an alternate expression for the friction factor as a dimensionless wall shear stress

$$f = \frac{8\tau_w}{\rho V^2} \tag{8.20}$$

Problem (1) pipe flow for real fluid

An oil with a viscosity of $\mu = 0.40\ \text{N}\cdot\text{s/m}^2$ and density $\rho = 900\ \text{kg/m}^3$ flows in a pipe of diameter $D = 0.020$ m. (a) What pressure drop, $p_1 - p_2$, is needed to produce a flowrate of $Q = 2.0 \times 10^{-5}\ \text{m}^3/\text{s}$ if the pipe is horizontal with $x_1 = 0$ and $x_2 = 10$ m? (b) How steep a hill, θ, must the pipe be on if the oil is to flow through the pipe at the same rate as in part (a), but with $p_1 = p_2$? (c) For the conditions of part (b), if $p_1 = 200$ kPa, what is the pressure at section $x_3 = 5$ m, where x is measured along the pipe?

Solution :

(a) If the Reynolds number is less than 2100 the flow is laminar and the equations derived in this section are valid. Since the average velocity is $V = Q/A = (2.0 \times 10^{-5}\ \text{m}^3/\text{s})/[\pi(0.020)^2\text{m}^2/4] = 0.0637$ m/s, the Reynolds number is $\text{Re} = \rho VD/\mu = 2.87 < 2100$. Hence, the flow is laminar and from Eq. 8.9 with $\ell = x_2 - x_1 = 10$ m, the pressure drop is

$$\Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4}$$

$$= \frac{128(0.40\ \text{N}\cdot\text{s/m}^2)(10.0\ \text{m})(2.0 \times 10^{-5}\ \text{m}^3/\text{s})}{\pi(0.020\ \text{m})^4}$$

or

$$\Delta p = 20{,}400\ \text{N/m}^2 = 20.4\ \text{kPa}$$

(b) If the pipe is on a hill of angle θ such that $\Delta p = p_1 - p_2 = 0$, Eq. 8.12 gives

$$\sin\theta = -\frac{128\mu Q}{\pi\rho g D^4}$$

or

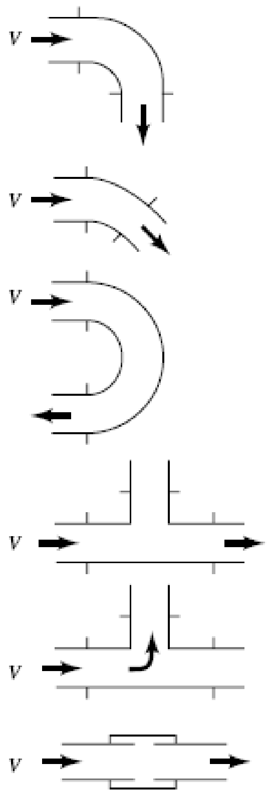
$$\sin\theta = \frac{-128(0.40\ \text{N}\cdot\text{s/m}^2)(2.0 \times 10^{-5}\ \text{m}^3/\text{s})}{\pi(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.020\ \text{m})^4}$$

$\theta = -13.34°$.

TABLE 8.2

Loss Coefficients for Pipe Components $\left(h_L = K_L \dfrac{V^2}{2g}\right)$ **(Data from Refs. 5, 10, 27)**

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210



Minor losses:

The most common method used to determine these head losses or pressure drops is to specify the *loss coefficient,* K_L which is defined as

$$K_L = \frac{h_L}{(V^2/2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

so that

$$\Delta p = K_L \tfrac{1}{2}\rho V^2$$

or

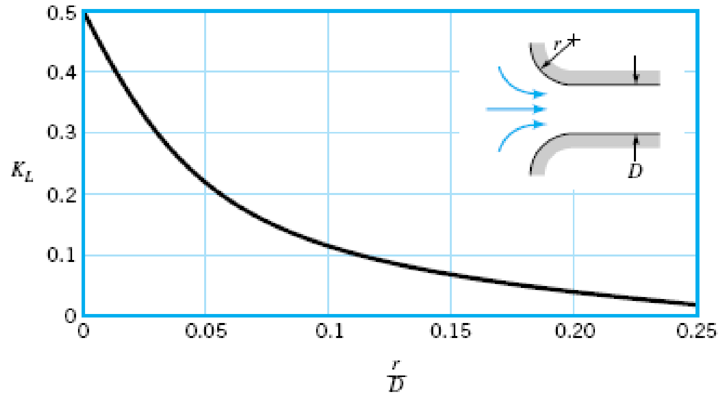
$$\boxed{h_L = K_L \frac{V^2}{2g}} \tag{8.36}$$

The pressure drop across a component that has a loss coefficient of $K_L = 1$ is equal to the dynamic pressure, $\rho V^2/2$.

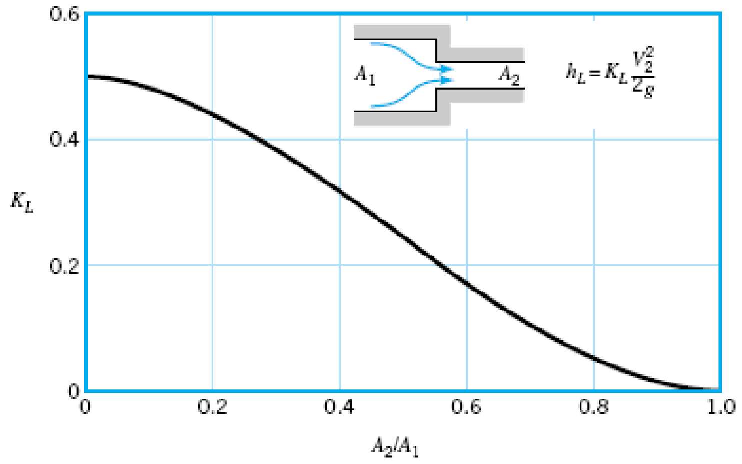
The actual value of K_L is strongly dependent on the geometry of the component considered. It may also be dependent on the fluid properties. That is,

$$K_L = \phi(\text{geometry, Re})$$

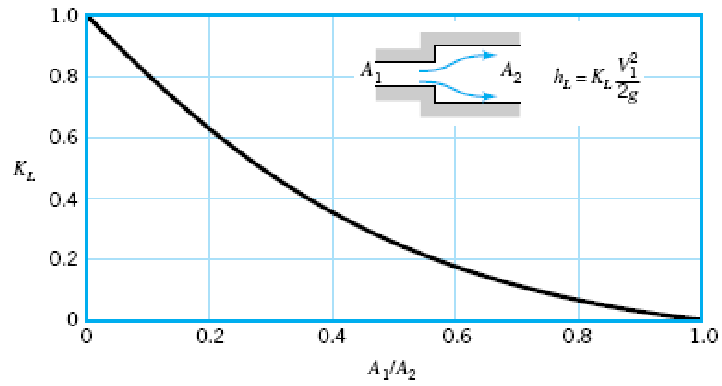
where $\text{Re} = \rho VD/\mu$ is the pipe Reynolds number. For many practical applications the Reynolds number is large enough so that the flow through the component is dominated by inertia effects, with viscous effects being of secondary importance. This is true because of the relatively large accelerations and decelerations experienced by the fluid as it flows along a rather curved, variable-area (perhaps even torturous) path through the component (see Fig. 8.21). In a flow that is dominated by inertia effects rather than viscous effects, it is usually found that pressure drops and head losses correlate directly with the dynamic pressure. This is the reason why the friction factor for very large Reynolds number, fully developed pipe flow is independent of the Reynolds number. The same condition is found to be true for flow through pipe components. Thus, in most cases of practical interest the loss coefficients for components are a function of geometry only, $K_L = \phi(\text{geometry})$.



■ **FIGURE 8.24**
Entrance loss coefficient as a function of rounding of the inlet edge (Ref. 9).



■ **FIGURE 8.26**
Loss coefficient for a sudden contraction (Ref. 10).



■ **FIGURE 8.27**
Loss coefficient for a sudden expansion (Ref. 10).

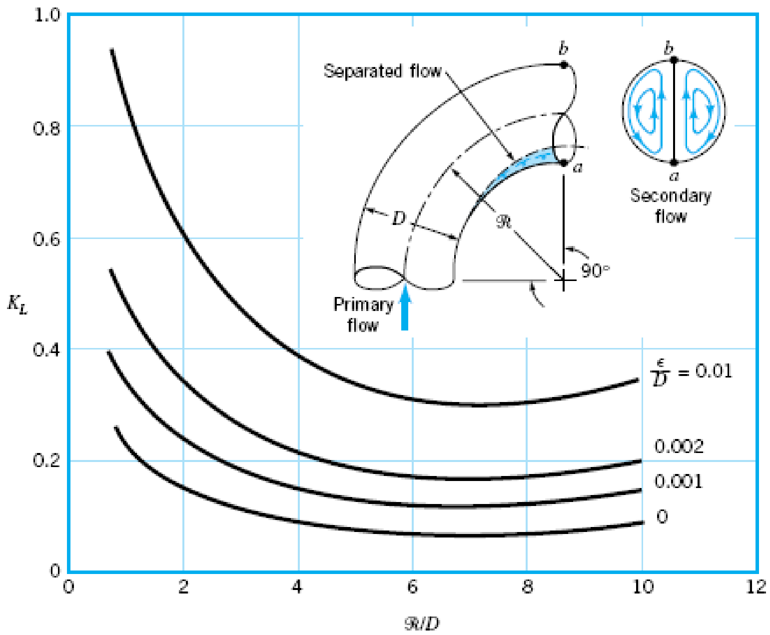


FIGURE 8.30 Character of the flow in a 90° bend and the associated loss coefficient (Ref. 5).

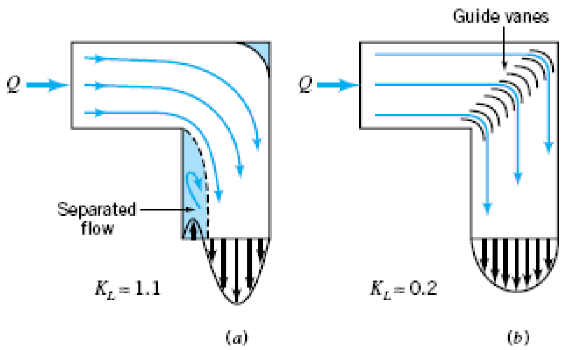


FIGURE 8.31 Character of the flow in a 90° mitered bend and the associated loss coefficient: (*a*) without guide vanes, (*b*) with guide vanes.

Example 1 *Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.*

Solution. Given :

Diameter of pipe $= 5\text{ cm} = 0.5\text{ m}$

Pressure, $p = 29.43\text{ N/cm}^2 = 29.43 \times 10^4\text{ N/m}^2$

Velocity, $v = 2.0\text{ m/s}$

Datum head, $z = 5\text{ m}$

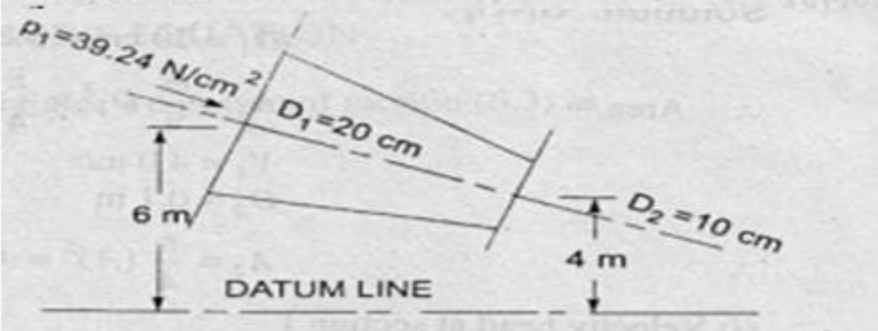
Total head = pressure head + kinetic head + datum head

Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30\text{ m}$ $\left\{\rho \text{ for water} = 1000\ \dfrac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204\text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 = 35.204\text{ m}$. **Ans.**

Example 2 *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.*



At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2, $D_2 = 0.10 \text{ m}$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow, $Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3/\text{s}$

Now $Q = A_1V_1 = A_2V_2$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or

$$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

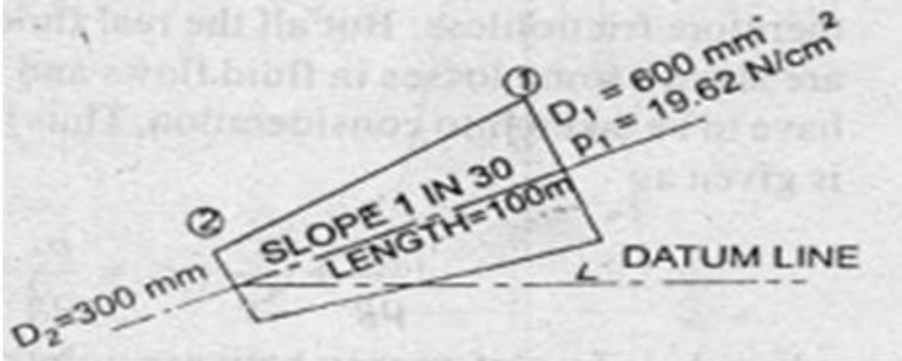
or

$$46.063 = \frac{p_2}{9810} + 5.012$$

$$\therefore \quad \frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$$\therefore \quad p_2 = 41.051 \times 9810 \text{ N/m}^2 = \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}$$

Example 3 The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm^2.



Solution. Given :

Length of pipe, $L = 100$ m

Dia. at the upper end, $D_1 = 600$ mm $= 0.6$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (.6)^2$
$= 0.2827$ m^2

p_1 = pressure at upper end
$= 19.62$ N/cm^2
$= 19.62 \times 10^4$ N/m^2

Dia. at lower end, $D_2 = 300$ mm $= 0.3$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (.3)^2 = 0.07068$ m

Q = rate of flow = 50 litres/s $= \frac{50}{1000} = 0.05$ m^3/s

Let the datum line is passing through the centre of the lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1 = \frac{1}{30} \times 100 = \frac{10}{3}$ m

Also we know $Q = A_1 V_1 = A_2 V_2$

$\therefore$ $V_1 = \frac{Q}{A} = \frac{0.05}{.2827} = 0.1768$ m/sec $= 0.177$ m/s

and $V_2 = \frac{Q}{A_2} = \frac{0.5}{.07068} = 0.7074$ m/sec $= 0.707$ m/s

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{.177^2}{2 \times 9.81} + \frac{10}{3} = \frac{p_2}{\rho g} + \frac{.707^2}{2 \times 9.81} + 0$$

or $$20 + 0.001596 + 3.334 = \frac{p_2}{\rho g} + 0.0254$$

or $$23.335 - 0.0254 = \frac{p_2}{1000 \times 9.81}$$

or $p_2 = 23.3 \times 9810$ N/m^2 $= 228573$ N/m^2 $=$ **22.857 N/cm^2. Ans.**

Application of Bernolli Equations

Example 4

The pitot and piezometer probes read the total and static pressures as shown in Fig. 3.14. Calculate the velocity V.

Bernoulli's equation provides

$$\frac{\cancel{V_2^2}}{\cancel{2}} + \frac{p_2}{\rho} + \cancel{g z_2} = \frac{V_1^2}{2} + \frac{p_1}{\rho} + \cancel{g z_1}$$

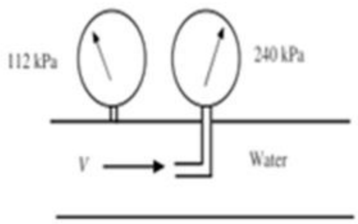


Figure 3.14

where point 2 is just inside the pitot tube. Using the information given, there results

$$\frac{240\,000}{1000} = \frac{V_1^2}{2} + \frac{112\,000}{1000} \qquad \therefore V_1 = 16\,\text{m/s}$$

Check the units on the first term of the above equation: $\frac{\text{N/m}^2}{\text{kg/m}^3} = \frac{(\text{kg}\cdot\text{m/s}^2)/\text{m}^2}{\text{kg/m}^3} = \frac{\text{m}^2}{\text{s}^2}$.

Problem 1

A pitot-static tube measures the total pressure p_T and the local pressure p in a uniform flow in a 4-cm-diameter water pipe. Calculate the flow rate if:

(*a*) $p_T = 1500$ mm of mercury and $p = 150$ kPa
(*b*) $p_T = 250$ kPa and $p = 800$ mm of mercury
(*c*) $p_T = 900$ mm of mercury and $p = 110$ kPa

Answer: (*a*) 10.01 m/s (*b*) 16.93 m/s (*c*) 4.49 m/s

Problem 2

Determine the velocity V in the pipe if the fluid in the pipe of Fig. 3.15 is:

(*a*) Atmospheric air and $h = 10$ cm of water
(*b*) Water and $h = 10$ cm of mercury
(*c*) Kerosene and $h = 20$ cm of mercury
(*d*) Gasoline and $h = 40$ cm of water

Sp.gravity for kerosen= 0.8 and for gasoline= 0.68

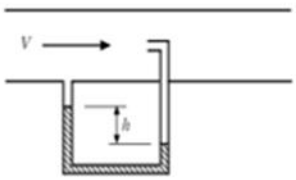


Figure 3.15

Answer 2
(*a*) 39.9 m/s (*b*) 4.97 m/s (*c*) 7.88 m/s (*d*) 1.925 m/s

Weirs

The notches are classified as:

1. According to the shape of the opening:
   (*a*) Rectangular notch,
   (*b*) Triangular notch,
   (*c*) Trapezoidal notch, and
   (*d*) Stepped notch.
2. According to the effect of the sides on the nappe:
   (*a*) Notch with end contraction.
   (*b*) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(*a*) According to the shape of the opening:
(*i*) Rectangular weir, (*ii*) Triangular weir, and
(*iii*) Trapezoidal weir (Cippoletti weir)

(*b*) According to the shape of the crest:
(*i*) Sharp-crested weir, (*ii*) Broad-crested weir,
(*iii*) Narrow-crested weir, and (*iv*) Ogee-shaped weir.

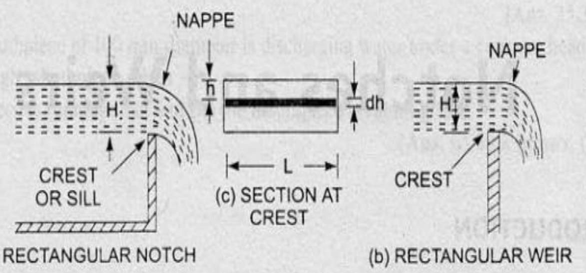


Fig. 8.1 *Rectangular notch and weir.*

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1.

Let H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h form the free surface of water as shown in Fig. 8.1(c).

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \qquad ...(i)$$

where C_d = Co-efficient of discharge.

The total discharge, Q, for the whole notch or weir is determined by integrating equation (i) between the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \cdot L \cdot \sqrt{2gh} \cdot dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1}\right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\, [H]^{3/2}. \qquad ...(8.1)$$

Problem 1:

Problem 8.2 *Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 litres/s. Take $C_d = 0.6$ and neglect end contractions.*

Solution. Given :

Length of weir, $L = 6$ m

Depth of water, $H_1 = 1.8$ m

Discharge, $Q = 2000$ lit/s $= 2\ m^3/s$

$C_d = 0.6$

Let H is height of water above the crest of weir, and H_2 = height of weir (Fig. 8.2)

The discharge over the weir is given by the equation (8.1) as

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\ H^{3/2}$$

or

$$2.0 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$= 10.623\ H^{3/2}$$

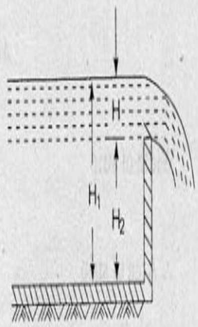


Fig. 8.2

$$\therefore \quad H^{3/2} = \frac{2.0}{10.623}$$

$$\therefore \quad H = \left(\frac{2.0}{10.623}\right)^{2/3} = 0.328 \text{ m}$$

$\therefore$ Height of weir, $H_2 = H_1 - H$

= Depth of water on upstream side $- H$

$= 1.8 - .328 =$ **1.472 m. Ans.**

Problem *The head of water over a rectangular notch is 900 mm. The discharge is 300 litres/s. Find the length of the notch, when $C_d = 0.62$.*

Solution. Given :

Head over notch, $H = 90$ cm $= 0.9$ m

Discharge, $Q = 300$ lit/s $= 0.3\ m^3/s$

$C_d = 0.62$

Let length of notch $= L$

Using equation (8.1), we have

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

or $$0.3 = \frac{2}{3} \times 0.62 \times L \times \sqrt{2 \times 9.81} \times (0.9)^{3/2}$$

$$= 1.83 \times L \times 0.8538$$

$\therefore$ $$L = \frac{0.3}{1.83 \times .8538} = .192 \text{ m} = \textbf{192 mm. Ans.}$$

DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

The expression for the discharge over a triangular notch or weir is the same. It is derived as :

Let H = head of water above the *V*- notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig. 8.3.

From Fig. 8.3 (*b*), we have

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$\therefore \quad AC = (H-h)\tan\frac{\theta}{2}$$

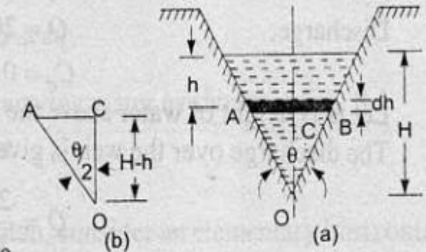


Fig. 8.3 *The triangular notch.*

Width of strip $= AB = 2AC = 2\,(H-h)\tan\frac{\theta}{2}$

$\therefore$ Area of strip $= 2\,(H-h)\tan\frac{\theta}{2}\times dh$

The theoretical velocity of water through strip $= \sqrt{2gh}$

$\therefore$ Discharge, dQ, through the strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2\,(H-h)\tan\frac{\theta}{2}\times dh \times \sqrt{2gh}$$

$$= 2C_d\,(H-h)\tan\frac{\theta}{2}\times\sqrt{2gh}\times gh$$

$\therefore$ Total discharge, Q is $\quad Q = \int_0^H 2C_d\,(H-h)\tan\frac{\theta}{2}\times\sqrt{2gh}\times dh$

$$= 2C_d \times \tan\frac{\theta}{2}\times\sqrt{2g}\int_0^H (H-h)h^{1/2}\,dh$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15}C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan\frac{\theta}{2} = 1$$

Discharge $$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$= 1.417\, H^{5/2}.$$

Problem 3: *Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.*

Solution. Given :

For rectangular weir, length, $L = 1$ m

Depth of water, $H = 150$ mm $= 0.15$ m

$C_d = 0.62$

For triangular weir, $\theta = 90°$

$C_d = 0.59$

Let depth over triangular weir $= H_1$

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 1.0 \times \sqrt{2 \times 9.81} \times (.15)^{3/2} \text{ m}^3/\text{s} = 0.10635 \text{ m}^3/\text{s}$$

The same discharge passes through the triangular right-angled weir. But discharge, Q, is given by equation (8.2) for a triangular weir as

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$\therefore \quad 0.10635 = \frac{8}{15} \times .59 \times \tan\frac{90}{2} \times \sqrt{2g} \times H_1^{5/2} \quad \{\because \theta = 90° \text{ and } H = H_1\}$$

$$= \frac{8}{15} \times .59 \times 1 \times 4.429 \times H_1^{5/2} = 1.3936\, H_1^{5/2}$$

$$\therefore \quad H_1^{5/2} = \frac{0.10635}{1.3936} = 0.07631$$

$$\therefore \quad H_1 = (.07631)^{0.4} = \mathbf{0.3572 \text{ m. Ans.}}$$

Stepped Notch

Problem 8.8 *Fig. 8.7 shows a stepped notch. Find the discharge through the notch if C_d for all section = 0.62.*

Solution. Given :

$$L_1 = 40 \text{ cm}, L_2 = 80 \text{ cm},$$
$$L_3 = 120 \text{ cm}$$
$$H_1 = 50 + 30 + 15 = 95 \text{ cm},$$
$$H_2 = 80 \text{ cm}, H_3 = 50 \text{ cm},$$
$$C_d = 0.62$$

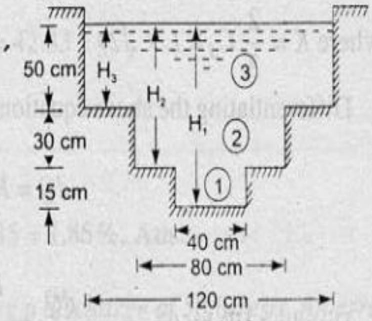


Fig. 8.7

Total discharge, $Q = Q_1 + Q_2 + Q_3$

where $$Q_1 = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 40 \times \sqrt{2 \times 981} \times [95^{3/2} - 80^{3/2}]$$

$$= 732.26[925.94 - 715.54] = 154067 \text{ cm}^3/\text{s} = 154.067 \text{ lit/s}$$

$$Q_2 = \frac{2}{3} \times C_d \times L_2 \times \sqrt{2g} \times [H_2^{3/2} - H_3^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 80 \times \sqrt{2 \times 981} \times [80^{3/2} - 50^{3/2}]$$

$$= 1464.52[715.54 - 353.55] \text{ cm}^3/\text{s} = 530141 \text{ cm}^3/\text{s} = 530.144 \text{ lit/s}$$

and $$Q_3 = \frac{2}{3} \times C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 120 \times \sqrt{2 \times 981} \times 50^{3/2} = 776771 \text{ cm}^3/\text{s} = 776.771 \text{ lit/s}$$

$$\therefore \quad Q = Q_1 + Q_2 + Q_3 = 154.067 + 530.144 + 776.771$$

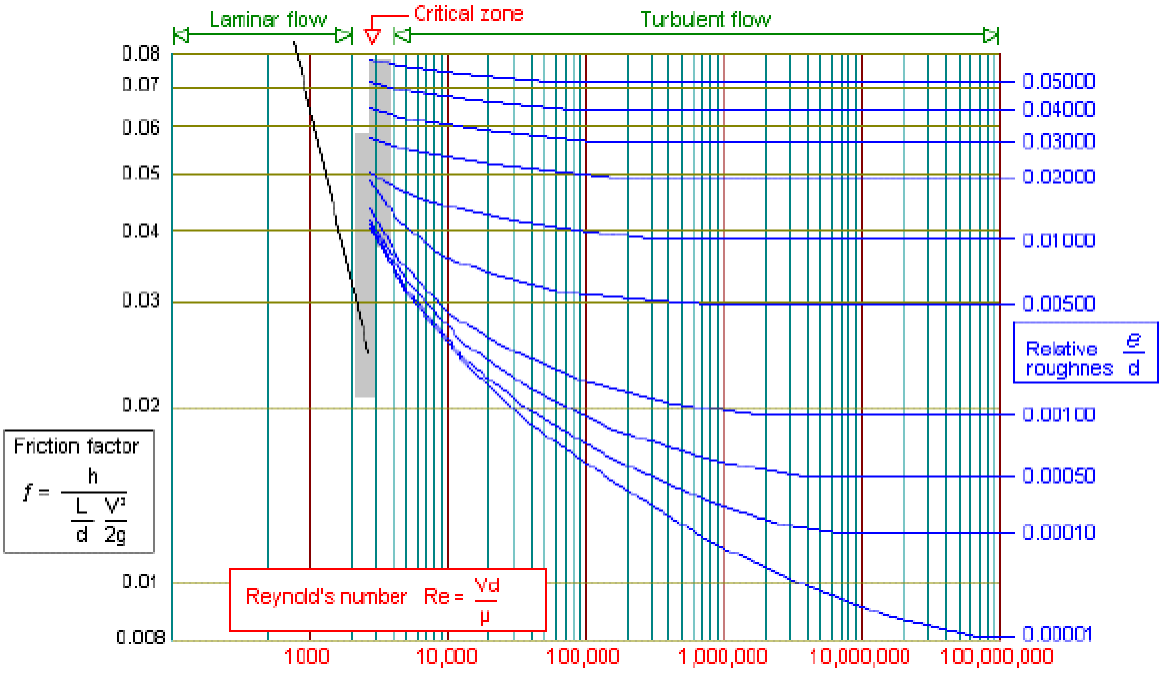
$$= 1460.98 \text{ lit/s. Ans.}$$

Friction Factor Calculations

The Darcy-Weisbach equation, for calculating the friction loss in a pipe, uses a dimensionless value known as the friction factor (also known as the Darcy-Weisbach friction factor or the Moody friction factor) and it is four times larger than the Fanning friction factor.

Friction Factor Chart / Moody Chart

The friction factor or Moody chart is the plot of the relative roughness (e/D) of a pipe against the Reynold's number. The blue lines plot the friction factor for flow in the wholly turbulent region of the chart, while the straight black line plots the friction factor for flow in the wholly laminar region of the chart.



In 1944, LF Moody plotted the data from the Colebrook equation and the resulting chart became known as ***The Moody Chart*** or sometimes the Friction Factor Chart. It was this chart which first enabled the user to obtain a reasonably accurate friction factor for turbulent flow conditions, based on the Reynolds number and the Relative Roughness of the pipe.

Friction Factor for Laminar Flow

The friction factor for laminar flow is calculated by dividing 64 by the Reynold's number.

Friction factor (for laminar flow) = 64 / Re

Critical Flow Condition

When flow occurs between the Laminar and Turbulent flow conditions (Re 2300 to Re 4000) the flow condition is known as critical and is difficult to predict. Here the flow is neither wholly laminar nor wholly turbulent. It is a combination of the two flow conditions.

Friction Factor for Turbulent Flow

The friction factor for turbulent flow is calculated using the Colebrook-White equation:

Colebrook-White Equation

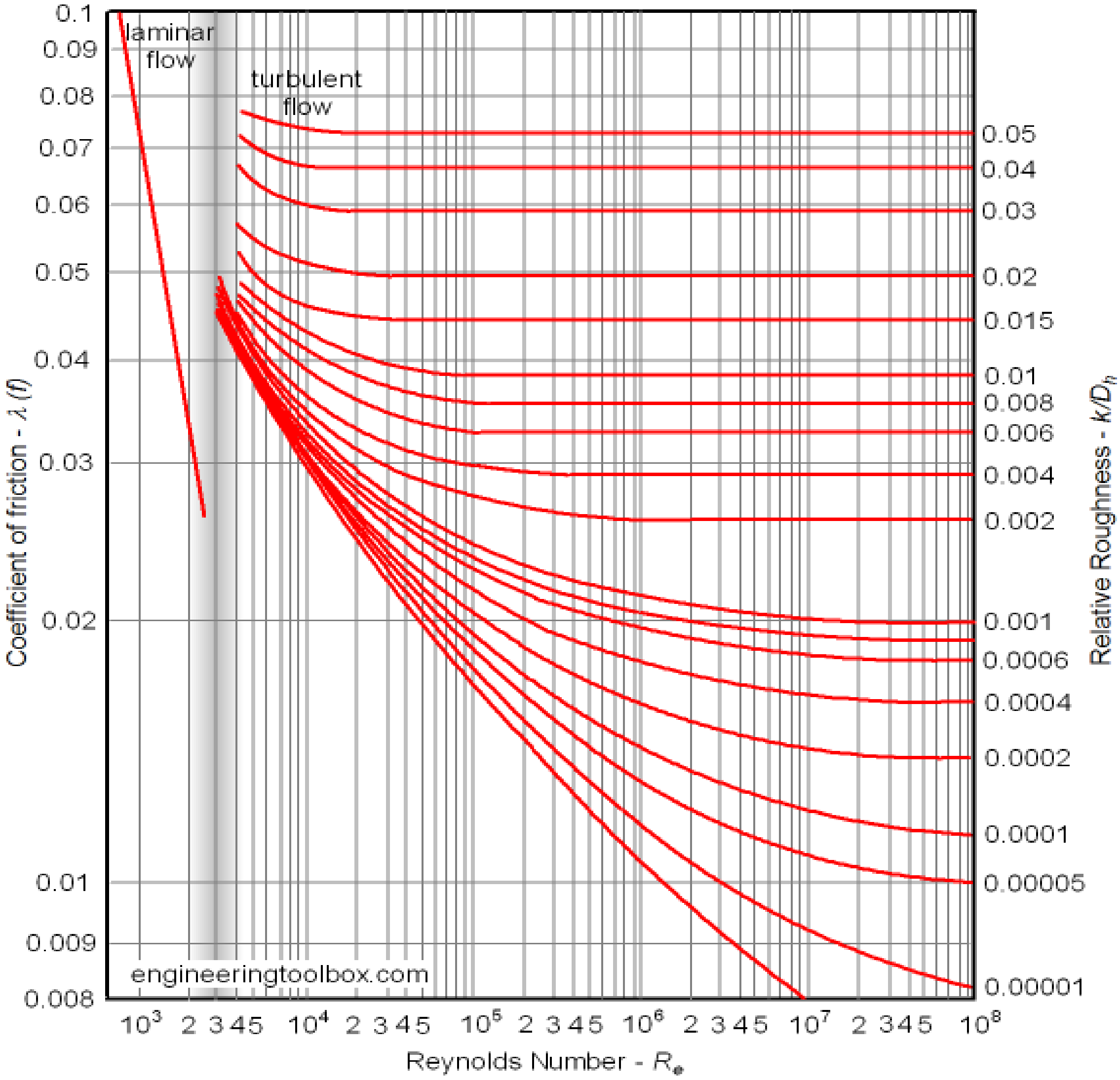
$$1/\sqrt{f} = 1.14 - 2\log_{10}\left(\frac{e}{D} + \frac{9.35}{\mathrm{Re}\sqrt{f}}\right) \quad \text{for Re} > 4000$$

Due to the implicit formation of the Colebrook-White equation, calculation of the friction factor requires an iterative solution via numerical methods.

The friction factor is then used in the Darcy-Weisbach formula to calculate the fluid frictional loss in a pipe.

SI based Moody Diagram

The Moody friction factor - *λ (or f)* - is used in the Darcy-Weisbach major loss equation. The coefficient can be estimated with the diagram below:



If the flow is transient - *2300 < Re < 4000* - the flow varies between laminar and turbulent flow and the friction coefiicient is not possible to determine. The friction factor can usually be interpolated between the laminar value at*Re = 2300* and the turbulent value at *Re = 4000*.

Example - SI based Friction Factor

For a PVC pipe with absolute roughness *k = 0.0015* 10^{-3} *(m)*, hydraulic diameter $D_h = 0.1$ *(m)* and Reynolds number $Re = 10^7$ - the relative rougness can be calculated as

$k/R_h = (0.0015\ 10^{-3}\ m) / (0.02\ m)$

$= \underline{0.000075}$

From the diagram above, with the relative rougness and the Reynolds number - the friction factor can be estimated to aprox. 0.011.

Example 1.1(Use of Moody Diagram to find frition factor): A commercial steel pipe, 1.5 m in diameter, carries a 3.5 m^3/s of water at 20^0C. Determine the friction factor and the flow regime (i.e. laminar-critical; turbulent-transitional zone; turbulent-smooth pipe; or turbulent-rough pipe)

Solution:

To determine the friction factor, relative roughness and the Reynolds number should be calculated.

For commercial steel pipe, roughness heigth (e) = 0.045 mm

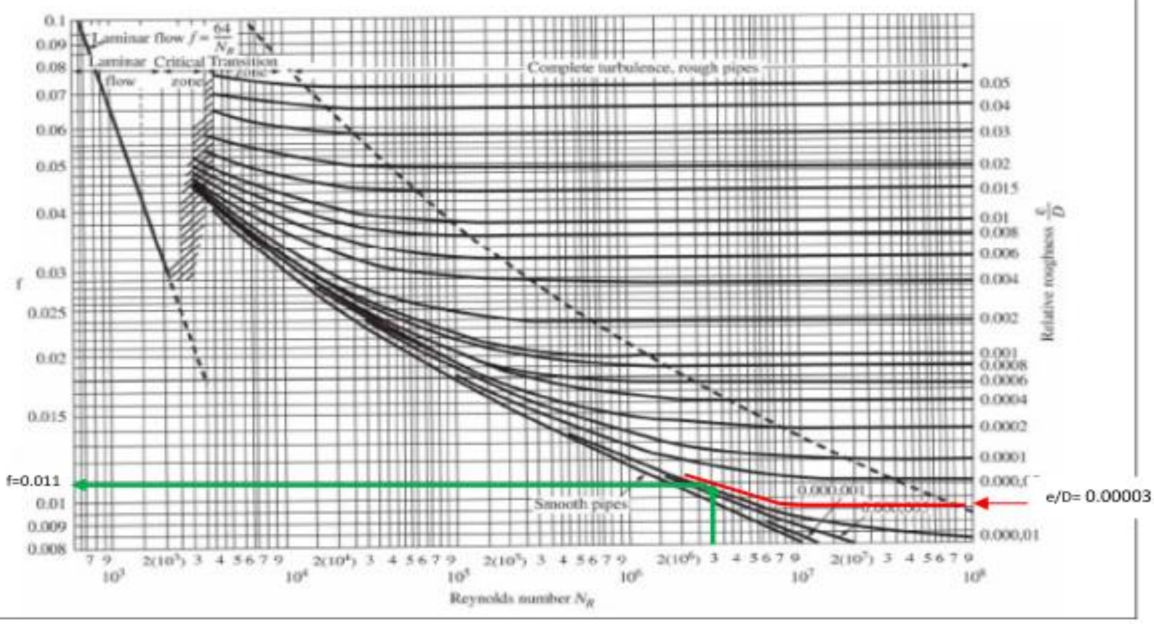
Relative roughness (e/D) = 0.045 mm/1500 mm = 0.00003

$V= Q/A=(3.5\ m^3/s)/[(\pi/4)(1.5\ m)^2]=1.98\ m/s$

$N_R=DV/\nu=[(1.5\ m)(1.98\ m/s)]/(1.00\text{x}10^{-06}\ m^2/s)=2.97\text{x}10^6$

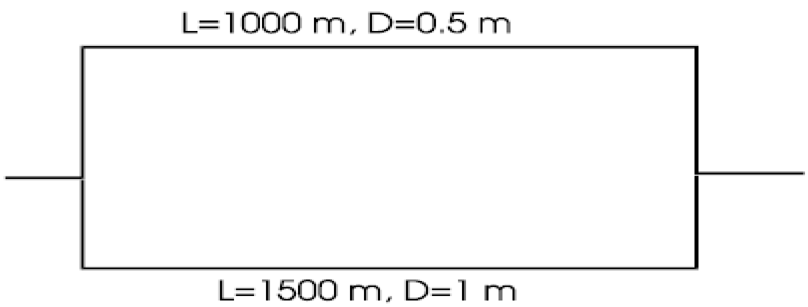
From Moody Diagram, f=0.011

The flow is turbulent-transitional zone.



Ex : (1)

A piping system consists of parallel pipes as shown in the following diagram.E One pipe has an internal diameter of 0.5 m and is 1000 m long. The other pipe has an internal diameter of 1 m and is 1500 m long. Both pipes are made of cast iron (ks = 0.26 mm). The pipes are transporting water at 20oC (ρ = 1000 kg/m3, ν = 10−6 m2/s). The total flow rate is 4 m3/s. Find the flow rate in each pipe and the pressure drop in the system. There is no elevation change. Neglect minor losses.



Solution

Designate the 1000-m pipe as pipe (1) and the other as pipe (2). The pressure drop along each path is the same, so

$$\Delta p = f_1 \frac{L_1}{D_1} \rho \frac{V_1^2}{2} = f_2 \frac{L_2}{D_2} \rho \frac{V_2^2}{2}$$

So, the velocity ratio between the two pipes is

$$\begin{aligned} \frac{V_2}{V_1} &= \sqrt{\frac{f_1}{f_2} \frac{L_1}{L_2} \frac{D_2}{D_1}} \\ &= \sqrt{\frac{1000}{1500} \times \frac{1.0}{0.5}} \sqrt{\frac{f_1}{f_2}} \\ &= 1.15 \sqrt{\frac{f_1}{f_2}} \end{aligned}$$

Since the total flow rate is 4 m^3/s,

$$\begin{aligned} V_1 A_1 + V_2 A_2 &= 4 \ \mathrm{m}^3/\mathrm{s} \\ V_1 \left(\frac{\pi}{4} \times 0.5^2 \right) + V_2 \left(\frac{\pi}{4} \times 1^2 \right) &= 4 \\ 0.196 V_1 + \frac{\pi}{4} \times 1.15 \times V_1 \sqrt{\frac{f_1}{f_2}} &= 4 \end{aligned}$$

or

$$V_1\left(0.196+0.903\sqrt{\frac{f_1}{f_2}}\right)=4$$

The relative roughness of pipe 1 is 0.26/500=0.00052 and for pipe 2, 26/1000=0.00026. We do not know the friction factors because they depend on the Reynolds number which, in turn, depends on the velocity. An iterative solution is necessary. A good initial guess is to use the friction factor for a fully rough pipe (limit at high Reynolds number). From the Moody diagram (Fig. 10.8) for pipe 1, take $f_1 = 0.017$ and for pipe 2, $f_2 = 0.0145$. Solving for V_1

$$\begin{aligned} V_1 &= \frac{4}{0.196+0.903\sqrt{\frac{0.017}{0.0145}}} \\ &= 3.41 \text{ m/s} \end{aligned}$$

and

$$\begin{aligned} V_2 &= 1.15\times 3.41\times\sqrt{\frac{0.017}{0.0145}} \\ &= 4.25 \text{ m/s} \end{aligned}$$

The Reynolds numbers are

$$\begin{aligned} \text{Re}_1 &= \frac{V_1 D_1}{\nu} = \frac{3.41\times 0.5}{10^{-6}} = 1.71\times 10^6 \\ \text{Re}_2 &= \frac{V_2 D_2}{\nu} = \frac{4.25\times 1.0}{10^{-6}} = 4.25\times 10^6 \end{aligned}$$

From the Moody diagram, the corresponding friction factors are

$$f_1 = 0.0172 \qquad f_2 = 0.0145$$

Because these are essentially the same as the initial guesses, further iterations are not necessary. The flow rates in each pipe are

$$\begin{aligned} Q_1 &= A_1V_1 = 0.196\times 3.41 = \underline{\underline{0.668 \text{ m}^3/\text{s}}} \\ Q_2 &= A_2V_2 = 0.785\times 4.25 = \underline{\underline{0.334 \text{ m}^3/\text{s}}} \end{aligned}$$

The pressure drop is

$$\begin{aligned} \Delta p &= f_1\frac{L_1}{D_1}\rho\frac{V_1^2}{2} \\ &= 0.0172\times\frac{1000}{0.5}\times 1000\times\frac{3.41^2}{2} \\ &= 2\times 10^5 \text{ Pa} = \underline{\underline{200 \text{ kPa}}} \end{aligned}$$

Ex:(2)

An old pipe 2 m in diameter has a roughness of $\epsilon = 30$ mm. A 12-mm-thick lining would reduce the roughness to $\epsilon = 1$ mm. How much would pumping costs be reduced per kilometer of pipe for water at 20 °C with discharge of 6 m^3/s? The pumps are 75 percent efficient, and the cost of energy is \$1 per 72 MJ.

$$v_1 = Q/A_1 = 6/[(\pi)(2)^2/4] = 1.910 \text{ m/s} \qquad N_R = dv/\nu$$

$$(N_R)_1 = (2)(1.910)/(1.02 \times 10^{-6}) = 3.75 \times 10^6 \qquad \epsilon_1/d_1 = (0.030)/2 = 0.015$$

From Fig. A-5, $f_1 = 0.044$.

$$d_2 = [2 - (2)(0.012)] = 1.976 \text{ m} \qquad v_2 = Q/A_2 = 6/[(\pi)(1.976)^2/4] = 1.957 \text{ m/s}$$

$$(N_R)_2 = (1.976)(1.957)/(1.02 \times 10^{-6}) = 3.79 \times 10^6 \qquad \epsilon_2/d_2 = (0.001)/1.976 = 0.000506$$

$$f_2 = 0.017 \qquad h_f = (f)(L/d)(v^2/2g)$$

$$(h_f)_1 = 0.044[1000/2]\{1.910^2/[(2)(9.807)]\} = 4.902 \text{ m}$$

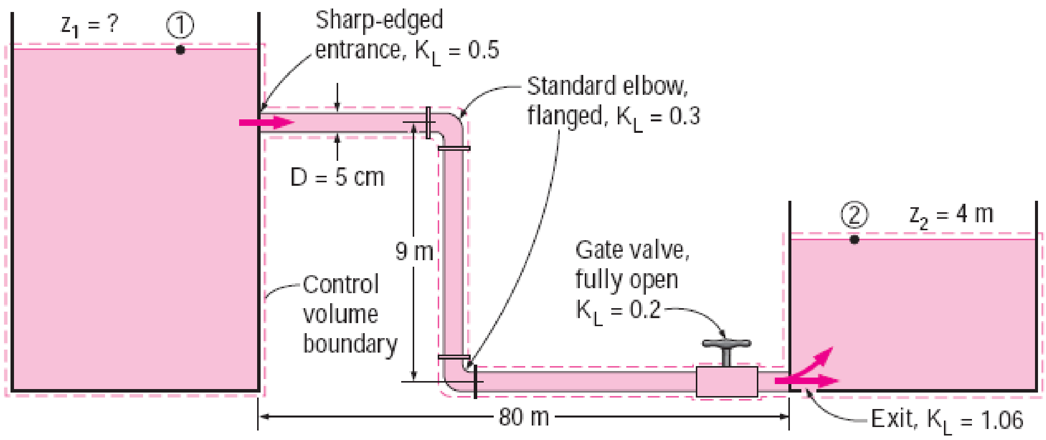
$$(h_f)_2 = 0.017[1000/1.976]\{1.957^2/[(2)(9.807)]\} = 1.680 \text{ m}$$

$$\text{Saving in head} = 4.092 - 1.680 = 2.412 \text{ m} \qquad P = Q\gamma h_f/\eta = (6)(9.79 \times 10^3)(2.412)/0.75 = 0.1889 \text{ MJ/s}$$

$$\text{Savings per year} = (0.1889)[(365)(24)(3600)]/72 = \$82\,738$$

EXAMPLE 1: Gravity-Driven Water Flow in a Pipe

Water at 10°C flows from a large reservoir to a smaller one through a 5-cmdiameter cast iron piping system, as shown in Fig. below. Determine the elevation z_1 for a flow rate of 6 L/s.



Note : V^o= Q = flow rate

SOLUTION The flow rate through a piping system connecting two reservoirs is given. The elevation of the source is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The elevations of the reservoirs remain constant. **3** There are no pumps or turbines in the line.

Properties The density and dynamic viscosity of water at 10°C are $\rho = 999.7\ \text{kg/m}^3$ and $\mu = 1.307 \times 10^{-3}\ \text{kg/m} \cdot \text{s}$. The roughness of cast iron pipe is $\varepsilon = 0.00026$ m.

Analysis The piping system involves 89 m of piping, a sharp-edged entrance ($K_L = 0.5$), two standard flanged elbows ($K_L = 0.3$ each), a fully open gate valve ($K_L = 0.2$), and a submerged exit ($K_L = 1.06$). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are zero ($V_1 = V_2 = 0$), the energy equation for a control volume between these two points simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \overbrace{\frac{V_1^2}{2g}}^{0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \overbrace{\frac{V_2^2}{2g}}^{0} + z_2 + h_L \quad \rightarrow \quad z_1 = z_2 + h_L$$

where

$$h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2/4} = 3.06\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(3.06\ \text{m/s})(0.05\ \text{m})}{1.307 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} = 117{,}000$$

The flow is turbulent since Re > 4000. Noting that $\varepsilon/D = 0.00026/0.05 = 0.0052$, the friction factor can be determined from the Colebrook equation (or the Moody chart),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives $f = 0.0315$. The sum of the loss coefficients is

$$\begin{aligned}\sum K_L &= K_{L,\text{entrance}} + 2K_{L,\text{elbow}} + K_{L,\text{valve}} + K_{L,\text{exit}} \\ &= 0.5 + 2 \times 0.3 + 0.2 + 1.06 = 2.36\end{aligned}$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} = \left(0.0315\frac{89\ \text{m}}{0.05\ \text{m}} + 2.36\right)\frac{(3.06\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 27.9\ \text{m}$$

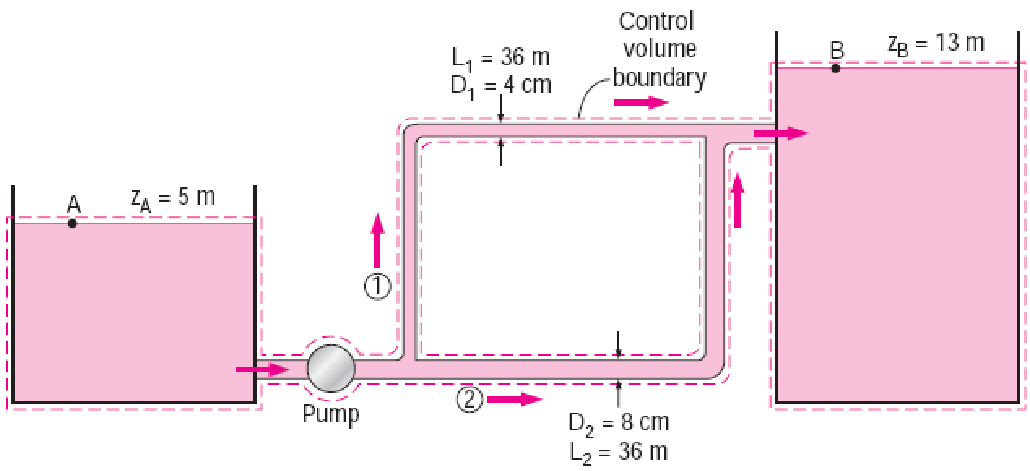
$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9\ m}$$

Therefore, the free surface of the first reservoir must be 31.9 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

EXAMPLE 2: **Pumping Water through Two Parallel Pipes**

Water at 20°C is to be pumped from a reservoir (zA ! 5 m) to another reservoir at a higher elevation (zB ! 13 m) through two 36-m-long pipes connected in parallel, as shown in Fig. 8–47. The pipes are made of commercial steel, and the diameters of the two pipes are 4 and 8 cm. Water is to be pumped by a 70 percent efficient motor–pump combination that draws 8 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.

SOLUTION The pumping power input to a piping system with two parallel pipes is given. The flow rates are to be determined.



Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** The minor losses and the head loss in pipes other than the parallel pipes are said to be negligible. **5** Flows through both pipes are turbulent (to be verified).

Properties The density and dynamic viscosity of water at 20°C are $\rho = 998\ \text{kg/m}^3$ and $\mu = 1.002 \times 10^{-3}\ \text{kg/m}\cdot\text{s}$. The roughness of commercial steel pipe is $\varepsilon = 0.000045$ m.

Analysis This problem cannot be solved directly since the velocities (or flow rates) in the pipes are not known. Therefore, we would normally use a trial-and-error approach here. However, nowadays equation solvers such as EES are widely available, and thus we will simply set up the equations to be solved by an equation solver. The useful head supplied by the pump to the fluid is determined from

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump, u}}}{\eta_{\text{pump-motor}}} \quad \rightarrow \quad 8000\ \text{W} = \frac{(998\ \text{kg/m}^3)\dot{V}(9.81\ \text{m/s}^2)h_{\text{pump, u}}}{0.70} \tag{1}$$

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{\text{atm}}$) and that the fluid velocities at both points are zero ($V_A = V_B = 0$), the energy equation for a control volume between these two points simplifies to

$$\cancel{\frac{P_A}{\rho g}} + \alpha_A \cancelto{0}{\frac{V_A^2}{2g}} + z_A + h_{\text{pump, u}} = \cancel{\frac{P_B}{\rho g}} + \alpha_B \cancelto{0}{\frac{V_B^2}{2g}} + z_B + h_L \rightarrow h_{\text{pump, u}}$$
$$= (z_B - z_A) + h_L$$

or

$$h_{\text{pump, u}} = (13 - 5) + h_L \tag{2}$$

where

$$h_L = h_{L,1} = h_{L,2} \tag{3)(4}$$

We designate the 4-cm-diameter pipe by 1 and the 8-cm-diameter pipe by 2. The average velocity, the Reynolds number, the friction factor, and the head loss in each pipe are expressed as

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi(0.04\ \text{m})^2/4} \tag{5}$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi(0.08\ \text{m})^2/4} \tag{6}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\ \text{kg/m}^3)V_1(0.04\ \text{m})}{1.002 \times 10^{-3}\ \text{kg/m}\cdot\text{s}} \tag{7}$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\ \text{kg/m}^3)V_2(0.08\ \text{m})}{1.002 \times 10^{-3}\ \text{kg/m}\cdot\text{s}} \tag{8}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right) \tag{9}$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{Re_2\sqrt{f_2}}\right)$$

$$\rightarrow \qquad \frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{Re_2\sqrt{f_2}}\right) \qquad (10)$$

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} \qquad \rightarrow \qquad h_{L,1} = f_1 \frac{36 \text{ m}}{0.04 \text{ m}} \frac{V_1^2}{2(9.81 \text{ m/s}^2)} \qquad (11)$$

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} \qquad \rightarrow \qquad h_{L,2} = f_2 \frac{36 \text{ m}}{0.08 \text{ m}} \frac{V_2^2}{2(9.81 \text{ m/s}^2)} \qquad (12)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \qquad (13)$$

This is a system of 13 equations in 13 unknowns, and their simultaneous solution by an equation solver gives

$\dot{V} = 0.0300 \text{ m}^3\text{/s}, \quad \dot{V}_1 = 0.00415 \text{ m}^3\text{/s}, \quad \dot{V}_2 = 0.0259 \text{ m}^3\text{/s}$

$V_1 = 3.30 \text{ m/s}, \quad V_2 = 5.15 \text{ m/s}, \quad h_L = h_{L,1} = h_{L,2} = 11.1 \text{ m}, \quad h_{pump} = 19.1 \text{ m}$

$Re_1 = 131{,}600, \quad Re_2 = 410{,}000, \quad f_1 = 0.0221, \quad f_2 = 0.0182$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.